Characterization of the effects of climate variation on land surface temperature and soil moisture through stochastic analysis of long term SSM/I observations over the Tibetan plateau

Ofwono Matthew
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Characterization of the effects of climate variation on land surface temperature and soil moisture through stochastic analysis of long term SSM/I observations over the Tibetan plateau

by

Ofwono Matthew

Thesis submitted to the International Institute for Geo-information Science and Earth Observation, in partial fulfilment of the requirements for the degree of Master of Science in Geo-information Science and Earth Observation, Specialisation: Surface Hydrology

Thesis Assessment Board

Chairman: Prof. Dr. Bob Su WRS Depart’ ITC - University of Twente
External examiner: Dr. Richard de Jeu Vrije Universiteit Amsterdam
Supervisor 1: Rogier Van der Velde WRS Depart’ ITC - University of Twente
Supervisor 2: Dr. Suhyb Salama WRS Depart’ ITC - University of Twente
Adviser: Lei Zhong WRS Depart’ ITC - University of Twente

INTERNATIONAL INSTITUTE FOR GEO-INFORMATION SCIENCE AND EARTH OBSERVATION
ENSCHENDE, THE NETHERLANDS
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Abstract

This study evaluates the temporal and spatial variability in land surface temperature (LST) and soil moisture over the Tibetan Plateau (TP) in the past 2 decades (1988-2008). The LST and soil moisture time series were derived from brightness temperatures ($T_B$) measured by the Special Sensor Microwave Imager (SSM/I). The retrieval is based on the $\tau$-$\omega$ model, whereby the horizontally (H) and vertically (V) polarized 19 GHz $T_B$ are utilized for the simultaneous inversion of soil moisture and the transmissivity. The comparison of the retrievals against a time series of LST and soil moisture measured over a period of 4-years gives an error of ±2.87 k and ±0.040 m$^3$m$^{-3}$ respectively. The western part of the TP is significantly colder than the eastern part due to a large elevation difference. The temporal variation of soil moisture over the TP follows the monsoon sequence. From November to March the soil moisture is almost zero. However as the monsoon season arrives in April the moisture content begins to rise reaching its maximum value by July and dissipates by October. Spatially, soil moisture is highest in the south and eastern parts of the TP. Image trend analysis revealed that, the entire TP experienced significant warming over the last two decades except the water bodies. However, the central TP experienced more significant positive trend in LST anomalies. A similar trend is observed over scattered areas in the east and northern part of TP, while the south and south eastern part of the TP experienced the least changes in temperature anomalies. The magnitude of warming over the TP is in the range of 0.2°C to 1.1°C/decade. Warming was more paramount at elevations over 3000m above sea level compared to areas with elevation below 3000m above sea level. The western and northern part of the TP experienced an increasing significant trend in mean annual soil moisture anomalies. While the eastern and south eastern part of the TP showed no trend in soil moisture anomalies. The soil moisture anomalies over the TP increased at the rate of 0.03 to 0.13% (volume/volume) per year. The annual trend was highest during the summer monsoon season than during the winter season.
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I would like to say thank you to my student advisor Lei Zhong and my colleagues who were patient enough with me when I “hijacked” five computers for nearly two months. I surely had to use those computers in order to complete my thesis within the stipulated time frame and I am sorry for any inconveniences that I could have caused to other students.
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1. Introduction

1.1. Background

Climate variability is one of the major global concerns due to its potential impact on socio-economic and political stability of the world. Understanding the changes and characterizing the natural variability of the global climate system has drawn the attention of the scientific community in the past few decades. Climate change indicators are physical variables that are strongly influenced by climatic conditions, such as land surface temperature and soil moisture (Latifovic and Pouliot 2007; Löscher, Retscher et al. 2008).

Land surface temperature (LST) and soil moisture are important factors in global change studies, energy balance, spatial and temporal dynamics of vegetation and have a role in controlling feedback mechanism of climate (Tan 2007; Raynolds, Comiso et al. 2008; Alec Sithole 2009). A change in soil moisture results in a change in the partitioning of net radiation into sensible heat flux and latent heat causing either a cooling or warming of the atmosphere. Soil moisture also influences runoff and is a limiting factor for plant growth.

Available data for the analysis of climate change studies are, in general, scarce and consist of point scale observations or simulated data. However, remote sensing plays a critical role in climate change studies by providing a synoptic information on the Earth’s environment at higher time frequency.

Numerous studies on climate variability, carried out over the past decade involved the use of climate proxies such as plant phenology, especially normalized difference vegetation index (NDVI) (Krishna Prasad, Badarinath et al. 2007; Nagai, Ichii et al. 2007; Tao, Yokozawa et al. 2008), derived from visible spectral bands. The visible part of the solar spectrum is, however, affected by atmospheric absorption and aerosol scattering, which require reliable atmospheric correction procedure. The use of microwave remote sensing overcomes this limitation of the visible band. Microwaves are longer and are able to penetrate clouds. Moreover, at these longer wavelengths emission is not only determined by the LST, but also the soil moisture content. Both land surface states are strongly affected by weather and can be used as indicators for climate change.

1.2. Problem statement

The effect of climate change will be most notable in vulnerable environments. One of such region is the Tibetan Plateau. The vast Tibetan landscape covers an area the size of Western Europe at an altitude of more than 4000 m above sea level. This huge obstacle in the centre of Asia has an important effect on guiding the high altitude jet streams over Asia and, as such, influences large scale weather systems (e.g. Asian Monsoon). The anomalies of surface heating over the Tibetan Plateau (TP), therefore, greatly influence atmospheric circulation in China, East Asia (EA) and even the Northern Hemisphere. The anomalies of surface heating field over TP might cause the abnormal atmospheric circulation in EA. Thus, it has great influences on the climate in China, especially the anomalies of summer precipitation in China. However, this investigation of the influence of climate
variation on land surface variables such as LST and soil moisture over the TP has received little attention.

1.3. **General Objective**

The general objective of this study is to evaluate the effects of possible climate variations on land surface temperature and soil moisture over the Tibetan plateau, using data retrieved from special sensor microwave imager (SSM/I) and stochastic methods.

1.3.1. **Specific Objectives**

The specific objectives of the study are:

- To retrieve land surface temperature and soil moisture of the study area from SSM/I;
- To evaluate the temporal variability of land surface temperature and soil moisture;
- To apply a stochastic method to identify the effects of climate variation on the retrieved soil moisture and LST.

1.4. **Research questions:**

- Are there any differences in the mean annual LST and soil moisture at 5% level of significance?
- What is the trend in LST and soil moisture over the TP over the last 20 years?
- What is (are) the parameter(s) and the order of the Auto Regressive Integrated Moving Average (ARIMA) model which describes the temporal LST and soil moisture variability over the TP?

1.5. **Hypothesis**

- The mean annual land surface temperature and the soil moisture between distinct years are statistically different at 95% confidence level;
- There is a statistically significant positive trend (increase) in land surface temperature and soil moisture over time;
- Both land surface temperature and soil moisture over the TP can be represented by ARIMA model of the same order for the entire area.

1.6. **Outputs**

- Mean LST and soil moisture time series maps;
- Time series plots of LST and soil moisture;
- Autocorrelation function (ACF) and partial autocorrelation function (PACF) plots of LST and soil moisture;
- ARIMA models for LST and soil moisture;
- Time series plots of LST and soil moisture residuals;
- Predicted LST and soil moisture time series.
1.7. Thesis structure

Chapter 1 provides a general introduction, problem statement, objectives, research questions, research outputs and a descriptive outline. Chapter 2 provides literature review on what has been done on climate change studies, soil moisture and LST retrievals and time series analysis. The study area and data set used in this study is described in chapter 3 while the methodologies and the theory behind the LST and soil moisture retrieval are explained in chapter 4. The results and discussions of LST and soil moisture retrieval are outlined in chapter 5. Chapter 6 evaluates the trend in LST and soil moisture time series. Chapter 7 discusses uncertainty and sensitivity analysis of the retrieval algorithm. Chapter 8 discusses the time series analysis. 9 provide a general summary and conclusion of this study followed by a list of used references.
2. Literature review

2.1. Global climate change

The industrial revolution brought with it an increased emission of greenhouse gases (GHG). The elevated concentration of GHG’s in the atmosphere enhanced the greenhouse effect resulting in an increase of 0.3 – 0.6°C in the global air temperature over the last century (Zamostny, Kukula et al. 1999; Matondo, Peter et al. 2004)

There is evidence that enhanced greenhouse effect not only increases the global air temperature, but may also have an impact on other climate variables. For example, Boyles and Raman,(2003) evaluated 49 years (1949 – 1998) of observed data (temperature and precipitation) to analyze climate variability over North Carolina using trend analysis of single point observations. The analysis revealed an increase in precipitation over the last 50 years during the winter seasons but a decrease during the summer seasons. The duration of warm season had also increased and the minimum temperature over the last 10 years of the study was higher than average minimum temperature, although it was not significantly different from the observed temperatures in the 1950s.

Based on digital elevation differences of three different dates and trend analysis of 50 years (1950 – 2000) of observed climate variables (monthly mean temperature and precipitation), Bown and Rivera, (2007) evaluated climate change and glacial behaviour over the Chilean lake district. The study found, there was a marked cooling of temperature over the study period at the lower altitude but a warming trend at higher elevation. Consequently, it was also apparent that there was glacial thinning over the study period, but the most significant finding was that the rate of thinning was 3 times faster between 1981 and 1998, leading to an annual drop rate in elevation by as much as 51m per annum. The overall change in elevation over the study period was -2.3m per year, well above the error margin of 0.6m. There was a general decrease in precipitation over the study area particularly during the last two decades of the study period.

Zhao et al. (2004) analyzed 30 years (1967 – 1997) of observed data (freezing depth, mean annual air temperature, ground surface temperature and annual precipitation) from 50 meteorological stations over the Tibetan plateau, using cluster analysis method for evidence of climate change. There was significant warming in the ground temperature than the air temperature and the warming was paramount in the warm season than in the cold season. However, considering the size of the Tibetan plateau, 50 meteorological stations are not representative of the area hence using remote sensing observations is more appropriate.

2.2. Remote sensing of land surface variables

It is almost 4 decades since the launch of Landsat-1 satellite into orbit. As such, sufficient data has been amassed to define climate normal at least in the visible and near infrared spectral domains. Also, via the special sensor microwave imager (SSM/I) on board the defence meteorological satellite program (DMSP), a time series of 21 years of observations in the microwave domain has been collected.
In comparison to wavelengths in the visible spectral domain of the electromagnetic spectrum, microwaves are less affected by atmospheric scattering, absorption by hydrometeors and can penetrate through clouds due to their long wavelengths. Moreover, microwave observations have been used in the past for the retrieval of physical quantities affected by climate variability such as LST and soil moisture.

### 2.2.1. Soil moisture and land surface temperature retrieval from passive microwave radiometers

Numerous algorithms have been developed for soil moisture and LST retrieval from passive microwave sensors. All these algorithms make use of the tau-omega model (Mo, Choudhury et al. 1982) for the retrieval of soil moisture and the relationship between emissivity and brightness temperature as basis for the retrieval of LST (McFarland, Miller et al. 1990; Drusch, Wood et al. 2001; Magagi and Kerr 2001; Morland, Grimes et al. 2001; Jackson, Hsu et al. 2002; Bindlish, Jackson et al. 2003; De Ridder 2003; Wen, Jackson et al. 2005; Gao, Wood et al. 2006).

Soil moisture retrieval algorithms based on the radiative transfer equation in which optical thickness is estimated from vegetation water content has been developed, for example, (Drusch, Wood et al. 2001; Jackson, Hsu et al. 2002; Bindlish, Jackson et al. 2003), used normalized difference vegetation index (NDVI) as a proxy for vegetation water content in their soil moisture retrieval algorithms.

Weng and Grody (1998) developed a land surface temperature retrieval algorithm based on the 19.35GHz and 22.2GHz SSM/I brightness temperature channels using a non linear algebra. Since the two frequencies are close to each other, variation in surface emissivity is minimized and there is a limited effect of atmospheric scattering and absorption by hydrometeors due to the relatively lower frequencies being used. Wen, Su et al (2003) used similar frequencies of TMI to retrieve land surface temperature based on the similar assumption.

### 2.3. Trend analysis of time series

de Beurs and Henebry (2005) proposed a framework for the analysis of long image time series involving two steps, that is; separation of mean values between periods which they referred to as step changes and image trend analysis within periods based on Mann – Kendall trend test rather than linear regression trend analysis method. The technique was then used to analyze land use changes over Kazakhstan using NDVI.

Vinnikov and Robock (2002) proposed a stochastic method to analyze trends in moments of climate indices in which observed trend in a climatic variable is deducted from the observed time series. A time series is then calculated for the resultant variables raised to different powers and a standard trend analysis is carried out for the generated time series. The method was used to analyze diurnal and seasonal surface air temperature (48 years of data 1951 -1999) from 9 meteorological stations distributed over the United States (Vinnikov, Robock et al. 2002)

Piwowar and Ledrew (2002) used an automated ARIMA process to analyze 9 years of sea ice concentration data, derived from scanning multichannel microwave radiometer (SMMR) while, Romilly (2005) applied ARIMA times series model to develop a forecasting model for global mean
temperature. In a similar vein, Kärner (2009) applied ARIMA time series to model long range temporal variability of total solar irradiance and surface air temperature time series.

Ford, Goranson et al (2005) used ARIMA time series model to estimate canopy transpiration based on sap flow obtained from Pinus taeda trees and multiple climatic variables that affect transpiration. The model explained 97% of the total variation in the data under both dry and wet conditions. The analysis revealed both an auto-regressive process (AR) and a moving average (MA) process.
3. Study area and data set

3.1. Description of the study area

The TP is situated in the western part of China between 80–105°E and 28–37°N. It is the highest plateau in the world and is characterized by mountain ranges with an average altitude of more than 4000 m above sea level, reaching the middle of the troposphere (Fu, Jiang et al. 2008). Figure 1 below shows different elevation classes over the Tibetan plateau.

![Digital elevation model of the Tibetan Plateau overlaid with locations of ARIMA model and Error analysis](image)

The area is associated with mean annual total precipitation of 240 mm, with most of the precipitation occurring during the Asian summer monsoon system from May to September. Only a fraction of the total annual rainfall is brought by the winter monsoon. The mean monthly air temperatures are -10.3°C in January to 15.0°C in July. The summer monsoon is replaced by the winter monsoon by mid-October (Wen, Su et al. 2003; Zhang, Cheng et al. 2003; Herzschuh, Kramer et al. 2009).

3.2. SSM/I Data set

The SSM/I has been measuring brightness temperature since 1987. It is on board of the defence meteorological satellite program (DMSP), it has seven-channels, four-frequencies, and it is an orthogonally polarized passive microwave radiometer. (Wen, Jackson et al. 2005). Table 1 below shows the characteristics of SSM/I.
Table 1: Basic characteristics of SSM/I

<table>
<thead>
<tr>
<th>Frequency (GHz)</th>
<th>Polarization</th>
<th>Spatial resolution (km)</th>
<th>Passes</th>
</tr>
</thead>
<tbody>
<tr>
<td>19.3</td>
<td>V &amp; H</td>
<td>69x43</td>
<td>both</td>
</tr>
<tr>
<td>22.2</td>
<td>V</td>
<td>50x40</td>
<td>both</td>
</tr>
<tr>
<td>37.0</td>
<td>V &amp; H</td>
<td>37x28</td>
<td>both</td>
</tr>
<tr>
<td>85.0</td>
<td>V &amp; H</td>
<td>15x13</td>
<td>both</td>
</tr>
</tbody>
</table>

The brightness temperature data was downloaded from the National Snow and Ice Data Centre (NSIDC) (Maslanik and Stroeve 2009). LST and soil moisture were then retrieved from the brightness temperatures for further analysis using the specified algorithms in chapter 4. The LST was derived from the V-polarized 37GHz frequency while soil moisture was derived from the 19GHz frequencies of the SSM/I instrument.

### 3.3. Measured land surface data

LST and soil moisture measured at 4 cm soil depth were obtained from the archives of the faculty of geo-information Science and Earth observation (ITC), Tibetan plateau field station measurements. The measured data used in this study were of Naqu station (31.3°N, 91.9°E). The temperature data were for the years 2005 through 2008 while soil moisture data measured at 4 cm depth were for the years 2005 through June 2008. However, there were missing data especially for the first half of 2005 and the second half of 2007 in the case of measured LST.
4. LST and soil moisture retrieval methods

4.1. Passive Microwave theory

Passive microwave instruments measure surface emission in the range of 1 to 100 cm wavelength. Under vegetated conditions, part of the emitted radiation is from vegetation while another component is from the soil. The emitted radiation is related to land surface temperature and land surface emissivity through Rayleigh – Jeans approximation of Planck’s radiation function. Microwave radiation is almost exclusively dependent on emissivity in a given frequency, land surface temperature and atmospheric transmission which is close to 1 in the microwave domain (Dash, Gottsche et al. 2002).

\[ T_B = \varepsilon T_S \]  \[1\]

Where \( T_B \) is the microwave brightness temperature at a given frequency, \( \varepsilon \) is the smooth surface emissivity and \( T_S \) is the thermodynamic land surface temperature of the emitting land surface.

According to Kirchoff’s law of radiation, absorptivity is equal emissivity at a given wavelength (Dash, Gottsche et al. 2002). It therefore follows that, since;

\[ \alpha + R = 1 \]  \[2\]

\[ R = 1 - \varepsilon \]  \[3\]

Where \( \alpha \) is the absorptivity, \( R \) is the smooth surface reflectivity and \( \varepsilon \) is the land surface emissivity.

The emitted land surface radiation is related to the relative permittivity (dielectric constant) of the soil through the Fresnel reflectivity function in the vertical and horizontal polarization (Ulaby, Moore et al. 1981)

\[ R_V = \left[ \frac{\varepsilon_r \cos \phi - (\varepsilon_r - \sin^2 \phi)^{0.5}}{\varepsilon_r \cos \phi + (\varepsilon_r - \sin^2 \phi)^{0.5}} \right]^2 \]  \[4\]

\[ R_H = \left[ \frac{\cos \phi - (\varepsilon_r - \sin^2 \phi)^{0.5}}{\cos \phi + (\varepsilon_r - \sin^2 \phi)^{0.5}} \right]^2 \]  \[5\]

Where \( R \) is the smooth surface reflectivity, \( V \) and \( H \) refers to vertical and horizontal polarizations, \( \varepsilon_r \) is the relative permittivity, \( \phi \) is the satellite zenith angle (degrees).

4.2. Dielectric constant and soil moisture

Retrieval of soil moisture from passive microwave signals is based on the large differences in the dielectric constant of water (\( \varepsilon_r = 80 \)) and the dielectric constant of dry soil (\( \varepsilon_r = 3.5 \)) and the resulting dielectric properties of a soil-water mixture (Wang and Schmugge 1980; Dobson, Ulaby et al. 1985). Soil being a heterogeneous material i.e. composed, rocks/minerals, air and water, its dielectric properties is a function of soil moisture, soil salinity, soil texture and the frequency of the emitted radiation (Wang and Schmugge 1980; Dobson, Ulaby et al. 1985; Hallikainen, Ulaby et al. 1985).
4.3. **Relationship between brightness temperature at 37GHz and LST**

LST is the skin temperature of the Earth’s surface and is a key component in various soil moisture retrieval algorithms (Magagi and Kerr 2001; Jackson, Hsu et al. 2002; Wen, Jackson et al. 2005; Sandells, Davenport et al. 2008; Liu, van Dijk et al. 2009). Previous studies have shown there is a linear relationship between the skin temperature and the brightness temperature at V-polarized 37 GHz channel (Owe, de Jeu et al. 2001; Holmes, de Jeu et al. 2009). In this study, measured LST was regressed on the brightness temperature for Naqu station for the period 2005 through July 2008 to derive land surface temperature for the simultaneous retrieval of soil moisture and vegetation transmissivity. Table 2 below summarizes the regression results, while figures 2 and 3 below demonstrate the goodness of fit of the model.

Table 2: Regression analysis of LST versus 37GHz v Pol. Brightness temperature

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Estimate</th>
<th>Standard error</th>
<th>t-value</th>
<th>P-value</th>
<th>Statistical significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>32.887</td>
<td>3.852</td>
<td>8.537</td>
<td>&lt; 0.001</td>
<td>***</td>
</tr>
<tr>
<td>Slope</td>
<td>0.937</td>
<td>0.015</td>
<td>62.471</td>
<td>&lt;0.001</td>
<td>***</td>
</tr>
</tbody>
</table>

Degree of freedom = 522, $R^2 = 0.882$, Residual standard error = 2.876k

Figure 2: LST versus 37GHz vertical polarization brightness temperature
Both the intercept and the slope are significantly different from zero at all levels of confidence and an examination of the normal Q-Q plot shows there is no need for data transformation. A high correlation coefficient and a relatively low residual standard error of 2.876K indicate the model has a good fit. The residuals are normally distributed with a mean of approximately zero. Therefore the land surface temperature over the TP was retrieved by using the relationship:

\[ LST = T_{B_{\text{37v}}}^{0.937} + 32.887 \]  

4.4 Retrieval of land surface temperature, Soil moisture and vegetation transmissivity

4.4.1 Soil moisture retrieval algorithm

Soil moisture retrieval algorithms are generally based on the radiative transfer equation e.g. (Jackson 1997; Jackson, Hsu et al. 2002; Bindlish, Jackson et al. 2003; Wen, Jackson et al. 2005). Differences between methods are embedded in the procedure for correcting the influence of vegetation on soil moisture retrieval. In this study a similar algorithm was used based on the dual polarized channel 19.35 GHz of SSM/I to solve vertical and horizontal smooth polarized surface reflectivity. Through an iterative process, volumetric soil moisture, vegetation transmissivity and land surface temperature were retrieved by minimizing the difference between satellite measured brightness temperature at

Figure 3: Normal probability plot of the residuals of LST vs 37GHz v.pol brightness temperature
19.35 GHz channel and simulated brightness temperature. An outline of the algorithm is shown in figure 4 below.

In this algorithm, two assumptions are made; the first assumption is that atmospheric effect on both 19GHz and 37GHz channels are negligible and the second assumption is that canopy temperature is equal to soil temperature. However, to ensure the second assumption holds true, the SSM/I data for the descending pass (morning hours) of F11, F13 and the ascending pass of F08 satellites were used in this study. This is because at this time of the day, thermodynamic temperature is relatively stable compared to during afternoon hours and therefore soil temperature can be assumed to be equal to canopy temperature (van de Griend and Owe 1994; Wen, Jackson et al. 2005)

4.4.2. Topp model

The commonly used dielectric mixing models for retrieving soil moisture are those of Dobson, Hallikainen and Wang-Schmugge (Wang and Schmugge 1980; Dobson, Ulaby et al. 1985; Hallikainen, Ulaby et al. 1985). However the drawback of these models is that they all require information on soil properties such as soil texture, soil porosity and wilting point which is not feasible to measure in an area the size of Tibetan plateau. On the other hand, the Topp model (G.C.Topp, J.L.Davis et al. 1980) does not require the input of soil texture, wilting point or vegetation water content. Moreover, de Jeu (2003) found that the influence of soil parameters on soil moisture retrieval were negligible. Consequently in this study the dielectric model of Topp was used. The model is given by;

$$\varepsilon = 3.03 + 9.3\theta + 146\theta^2 - 79.7\theta^3$$  \[7\]

Where \(\varepsilon\) is the dielectric constant and \(\theta\) is the volumetric soil moisture

4.4.3. Vegetation effect on emissivity

The vegetation cover affects emissivity in two ways, not only does it emit its own radiation into the atmosphere but it also scatters and absorbs radiation emitted by the soil layer. Vegetation, therefore, acts as an attenuating and emissive layer above the soil, and the vegetation scattering is a function of vegetation water content, geometric structure, and spatial distributions of stem and leaf components (Njoku and Chan 2006). If the vegetation cover is too thick, most of the radiation measured by the satellite sensor will be emanating from the vegetation as soil emission is attenuated (Wigneron, Calvet et al. 2003). The radiative transfer equation of Mo, commonly referred to as the tau – omega model (Mo, Choudhury et al. 1982) accounts for the influence of vegetation on emissivity and therefore on the retrieved soil moisture. The model represents upwelling surface radiation in terms of brightness temperature and is given by;

$$T_B = (\varepsilon \Gamma)T_s + (1 - \omega)(1 - \Gamma)T_c + (1 - \varepsilon)(1 - \omega)(1 - \Gamma)\Gamma T_c$$  \[8\]

Where \(\varepsilon\) is the land surface emissivity, \(\Gamma\) is the vegetation canopy transmissivity, \(\omega\) is the single scattering albedo and \(T_c \approx T_s\) is the LST.

The first term of equation 8 accounts for the emission by the soil layer corrected for vegetation attenuation, the second term accounts for the emission by the vegetation only while the last term...
accounts for the vegetation emitted radiation that interacts with the soil surface before being attenuated by the vegetation as it propagates into the atmosphere (Owe, de Jeu et al. 2001; Wigneron, Calvet et al. 2003).

The canopy transmissivity $\Gamma$ determines how transparent the vegetation is to the microwave emission and is defined in terms of canopy optical thickness $\tau$ and the angle of incidence $\theta$ of the emitted radiation (Van de Griend and Owe 1993; van de Griend and Owe 1994; van de Griend, Owe et al. 1996; Bindlish, Jackson et al. 2003). $\Gamma$ is defined as:

$$\Gamma = e^{-\frac{\tau}{\cos\theta}}$$  \[9\]

### 4.4.4. Effect of surface roughness on emissivity

A rough surface has a larger surface area than a smooth surface, consequently surface roughness increases land surface emissivity and therefore increased brightness temperature measured by the satellite sensor at a given soil moisture condition (Wigneron, Calvet et al. 2003). The roughness parameter $h$ is formulated as a function of the root mean square variation of surface height and the wave number which is a function of frequency (Choudhury, Schmugge et al. 1979; Wang and Choudhury 1981; Wang 1985; Njoku and Chan 2006). The empirical model of Wang and Choudhury (1981) was used in this study to correct land surface emissivity for surface roughness, because it accounts for depolarization caused by scattering of radiation between different surfaces. Smooth Surface reflectivity is given by:

$$R = 1 - \epsilon_{sur}$$  \[10\]

Where $\epsilon_{sur}$ is the surface emissivity

Surface reflectivity corrected for roughness is defined as:

$$R^p = \left[ (1 - Q)R_o^p + QR_o^q \right] e^{-k^2\delta^2\cos^2\varphi}$$  \[11\]

Where $Q$ is a parameter that increases with surface roughness, $R$ is the smooth surface reflectivity, $p$ and $q$ refer to orthogonal polarization, $\delta$ is the root mean square variation in surface height (cm), $\varphi$ is the satellite viewing zenith angle (degrees) and $k$ is the wave number given by $2\pi/\lambda$ (cm$^{-1}$). The parameter $Q$ characterises both $\sigma$ and the roughness correlation length. Kerr and Njoku (1990) estimated the parameter $Q$ from equation 12 below.

$$Q = 0.35[1 - e^{-0.66\delta^2f^2}]$$  \[12\]

Where $f$ is the frequency in GHZ

Figure 4 below is a schematic illustration of the soil moisture retrieval algorithm. Through the inversion of the tau - omega model, soil moisture and vegetation transmissivity are simultaneous retrieved. Initial soil moisture is used to compute the dielectric constant using Topp model. The resultant dielectric constant is then used to compute the smooth surface reflectivity in both the vertical and horizontal polarizations. The smooth surface reflectivity is then corrected for surface roughness using the method of Wang and Choudhury (1981). The reflectivity is then used to compute the
brightness temperatures. By minimizing the differences between the computed brightness temperatures and the brightness temperatures measured by the SSM/I sensor at 19GHz channel, the soil moisture and vegetation transmissivity are retrieved simultaneously.
Characterization of the effects of climate variation on land surface temperature and soil moisture through stochastic analysis of long term SSM/I observations over the Tibetan Plateau

\[ \text{Canopy Transmissivity } \Gamma \]

Land surface temperature \( \text{LST} = T_c = T_e \)

Soil moisture \( \Theta \)

\[ \text{Relative dielectric constant } \varepsilon_r = 3.03 + 9.3\theta + 146\theta^2 - 79.7\theta^3 \]

\[ Q = 0.35 \left[ 1 - e^{-0.666\sigma^2 f^2} \right] \]

Smooth surface Reflectivity

\[ R_H = \frac{\cos\varphi - (\varepsilon_r - \sin^2\varphi)^{0.5}^2}{\cos\varphi + (\varepsilon_r - \sin^2\varphi)^{0.5}^2} \]

\[ R_V = \frac{\varepsilon_r \cos\varphi - (\varepsilon_r - \sin^2\varphi)^{0.5}^2}{\varepsilon_r \cos\varphi + (\varepsilon_r - \sin^2\varphi)^{0.5}^2} \]

Observed SSM/I 19V&19H brightness temperature \( T_B \)

\[ T_B = \varepsilon \Gamma T_s + (1 - \omega)(1 - \Gamma)T_c + (1 - \varepsilon)(1 - \omega)(1 - \Gamma)\Gamma T_c \]

Figure 4: Schematic diagram of LST, soil moisture and vegetation transmissivity retrieval

Iteration till minimum root mean square error (RMSE) of \( T_B \)
In literature surface roughness parameter $h$ given by $\delta^2 k^2$, is thought to range between 0.0 and 0.4 (Owe, de Jeu et al. 2001). In this study, soil moisture over Naqu south station was compared to soil moisture retrieved using different values of surface roughness as shown in figure 5 below. A roughness parameter value of 0.0 cm yielded soil moisture in agreement with the ground measurements. The value of zero represents a flat surface which is characteristic of the TP and is consistent with finding from previous studies (Van de Griend and Owe 1993), as a result a roughness parameter value of 0.0 cm was adopted for the study area.

Figure 5: Influence of different values of surface roughness on retrieved soil moisture
4.4.5. Retrieval of soil moisture from frozen soils

Wegmüller (1990) studied the effect of freezing and thawing on the microwave signatures of bare soils and concluded that frozen soils behave like dry soils because the liquid water content is small and in the range of 0 to 0.05 cm\(^3\) cm\(^{-3}\). The LST model used to retrieve soil moisture in this study was derived from the relationship between surface temperature measured at 4cm soil depth and the brightness temperature at 37 GHz channel of SSM/I. During winter season, however, snow accumulates and soil freezes. Because frozen soils have higher emissivity than wet soils (Wegmüller 1990) due to the fact that water molecules in a frozen state are held tightly together to the extent that free rotation of water molecules is undermined and the surface temperature measured by the data logger is no longer at 4cm depth but rather 4cm plus snow depth, the soil moisture retrieved during the winter season is distorted as shown in the figure 6 above (day 0-120).

This is largely due to increased emissivity caused by freezing and the increased depth of surface emission. To account for this effect, a surface roughness parameter of 0.05 was used for this period of the year. The results of the calibration is summarized in Fig. 7 below which compares measured soil moisture to soil moisture retrieved with two sets of roughness parameter \(h\), i.e. 0.05 during freezing time of the year and 0.0 cm for the rest of the year. Panciera, Walker et al.(2009) proposed that, the dependence of surface roughness parameter on available soil moisture could be explained by the effect of volume scattering i.e. as available soil moisture diminishes, soil emission originates from deeper soil layers.
4.4.6. Effect of single scattering albedo on soil moisture retrieval

The single scattering albedo is a parameter which is a function of vegetation geometry and it directly affects the partitioning of the radiation emitted by the vegetation into scattering and absorption (Wen, Su et al. 2003). Several ranges of values are reported in literature e.g. van de Griend, Owe et al. (1996) reported a single scattering albedo of 0.06 to 0.12. Similar values of single scattering albedo were presented by Lee and Anagnostou (2004). In this study the single scattering albedo was varied from 0.03 to 0.07 to investigate the suitable value for soil moisture retrieval over the Tibetan plateau. A single scattering albedo of 0.05, retrieved soil moisture which matched the measured soil moisture at Naqu site for the year 2006, consequently single scattering albedo of 0.05, was used for soil moisture retrieval over the Tibetan plateau. Figure 8 below summarizes the comparison between measured soil moisture and soil moisture retrieved with varying values of single scattering albedo.
Figure 8: Effect of single scattering albedo on soil moisture retrieval (Naqu station 2006 data)
5. Results and discussions

The retrieved LST and soil moisture were compared to the ground measured LST and soil moisture for the periods 2005 through July 2008 as shown in figures 9 and 10 below. There is a good agreement between the retrieved land surface variable and ground measured variables.

Figure 9: comparison between measured LST and retrieved LST

Figure 10: comparison between retrieved soil moisture and measured soil moisture
Single scattering albedo is generally thought to be time invariant (van de Griend and Owe 1994; Wen, Su et al. 2003), it was therefore justifiable to use a constant representative value of the single scattering albedo. The roughness parameter of 0.0 cm is a reasonable value for the Tibetan plateau and is in agreement with a value obtained for a similar environment (van de Griend and Owe 1994), considering the fact that the study area is a plateau interspaced with mountain ranges and is a semi arid area. However, the effect of volume scattering during the freezing conditions should be taken into account.

The retrieved vegetation transmissivity showed seasonal variation with the highest value occurring during the peak of winter and the lowest value occurring during the peak of summer. This is expected because, during the peak of winter, vegetation on the Plateau becomes inactive and its biomass reduces, hence a higher transparency to microwave emission. On the other hand during the peak of summer nearly all the vegetation have full grown vegetation canopy, and therefore less transparent to the microwave emission. This is in agreement with the findings from previous studies of similar environment e.g. (Van de Griend and Owe 1993; van de Griend and Owe 1994). Figure 11 below shows the seasonal variation of vegetation transmissivity over the period from 2004 till 2008 on the Tibetan plateau.

![Retrieved vegetation transmissivity](image)

Figure 11: Retrieved vegetation transmissivity

The retrieval error for the land surface temperature was 0.00±2.87k, this is quite in agreement with the algorithm of (Wen, Su et al. 2003) over central Tibetan plateau, while the soil moisture retrieval error was 0.00±0.04m\(^3\)m\(^{-3}\). Figure 12 below summarizes the error distribution of retrieved LST and soil moisture.
The temporal variation of soil moisture over the Tibetan plateau follows the monsoon sequence. From November to March the soil moisture is almost zero. However, as the monsoon season arrives in April (Zhang, Cheng et al. 2003) the moisture content begins to rise reaching its maximum value by July and dissipates by October. As shown in figure 13a soil moisture is highest in the south and eastern parts of the TP. This is partially attributed to the fact that the south eastern part of the TP is relatively low compared to the western section. See figure 1 above. Since fluids seek areas of least resistance to flow, the Indian monsoon winds are directed towards the south east. Hence, clouds release most of their water in that region of the TP. Areas with elevation of more than 4000m above sea level (western part) are significantly colder than regions with elevation below 4000m above sea level as shown in figure 13b. On the Plateau, specifically the large water bodies exhibit low temperatures. This is in agreement with the findings of Wen, Su et al. (2003). The low temperatures in the western part of the Tibetan plateau coupled with the influence of Taklamakan desert in the North West account for the low soil moisture in the western region of the TP.
Figure 13: Spatial soil moisture (A) and LST (B) distribution over the Tibetan plateau (July 2004)
6. Evaluation of trend in the soil moisture and LST time series

Katz and Brown (1992), suggested that climate change is more evident as changes in variability rather than changes in averages. With a stack of 21 years of images, one of the retrievable measures of variability is the trend in soil moisture and temperature anomalies. A simple linear regression was fitted to the time series for every image pixel in the mean annual stack and the slope, standard error of the estimate and t-values of each trend line were assigned to corresponding pixels in the new soil moisture and temperature images (Liu and Chen 2000). Since monthly mean soil moisture and monthly mean temperature are autocorrelated, the trend analysis was done using soil moisture and temperature anomalies. The anomalies were computed by subtracting each, monthly mean soil moisture or temperature from the long term (21 year) monthly means (Hipel and Mcleod 1994).

Figure 14 below, show spatial patterns of simple linear trend in annual average soil moisture with the corresponding t-statistics. The critical t-value for a two tailed t-test is 2.09 at 5% level of significance. All t-values that fall outside of ± 2.09 are statistical significant at 95% confidence limits. Between 1987 and 2008, the western and northern part of the TP experienced an increasing significant soil moisture trend while the eastern and south eastern part of the TP showed no trend in soil moisture. This is in agreement with Zhao, Ping et al. (2004) who found that annual precipitation increased in the North western and the central part of the Tibetan plateau. The Taklamakan desert in the north western part of the TP experienced a decreasing significant soil moisture trend.

Further examination of the spatial patterns of linear trend in monthly average normalized soil moisture anomalies indicate that, the increasing trends occurred during the summer monsoon seasons, i.e. April through October, while the decreasing trends occurred during the off summer monsoon season i.e. November through April. Wide spread positive trend in normalized soil moisture anomalies occurred during the months of April through July.

While during the months of August through October, positive trend in normalized soil moisture anomalies were concentrated at the central part of the TP. This finding is in agreement with Liu, Wang et al. (2009) who demonstrated that two high elevation inland lakes at elevation of 4600m over
the central TP had increased in surface area by a maximum of 27.1% over an eight year period due to an increase in annual precipitation by 12.6% and increase in annual mean temperature by 0.41°C over the same period (1998-2005). A seasonal decomposition of the 21 year monthly soil moisture time series using loess procedure reveals a trend in soil moisture which is in agreement with the suggestion by Liu, Wang et al. (2009), that the changes in precipitation over the central TP was abrupt between 1996 and 2005 (figure 15 below).

Figure 15: 21 year monthly mean soil moisture trend over the central TP
Figure 16: Trend in mean monthly soil moisture during the monsoon season.
Figure 17: Normalized soil moisture anomaly magnitudes during the monsoon season
Figure 16 above shows that the average rate of soil moisture increment during the months of May through October (September and October are not shown) is 0.004 m$^3$ m$^{-3}$ which is four times the annual rate. This is due to the fact that soil moisture contributed by the months of November through February to the total annual average is negligible. While figure 17 above demonstrates that the order of magnitude of the soil moisture anomalies is similar for all the 6 months and is in the range of ±1.5.

Figure 18 below demonstrates that between 1987 and 2008, the entire TP experienced significant warming. However, the central TP experienced more significant positive trend in land surface temperature anomalies. A similar trend is observed over scattered areas in the east and northern part of TP while the south and south eastern part of the TP experienced the least changes in temperature anomalies. This is in agreement with the findings of (Liu and Chen 2000; Liu, Cheng et al. 2009) who found that warming over the TP increases with increasing elevation. Zhao, Ping et al. (2004) found that the south eastern part of the TP had the least warming between 1967 and 2000. It should be noted that central and western TP has a significantly higher elevation than the south eastern part.

The overall warming trend over the TP of 0.1 to 1.1°C/decade is consistent with the finding of Trenberth, Jones et al. (2007). However, the central TP has warmed at the rate of 1.1°C /decade, which is 0.35°C above the global average of 0.75°C/decade (Trenberth, Jones et al. 2007). When viewed on monthly basis, the eastern part of the TP experienced more pronounced warming during the months of October through March. This is consistent with the findings of Liu and Chen (2000) and Liu, Cheng (2009) who found that warming in the eastern TP was more pronounced during the winter season. On the other hand the central and northern part of the TP experienced more pronounced warming during the months of May through October which is in agreement with the findings of Zhao, Ping et al. (2004)
Figure 19: Comparison between warming during February and June

Figure 19 above shows that the annual rate of warming over the central TP was highest during the summer season at the rate of 1.3°C/decade while during the winter and spring season, the rate of warming was 0.41°C/decade. This is attributed to melting processes consuming much of the energy during the spring time and less energy spent on warming the surface.
7. Uncertainty and sensitivity analysis

7.1. Derivation of uncertainty matrix

The magnitude of errors caused by uncertainties in the input variables was computed using the error propagation analysis (deJeu 2003; Shi, Cheung et al. 2004; Puatanachokchai and Mikhail 2008). The brightness temperature errors were obtained from the SSM/I users guide (Hollinger, R. Lo et al. 1987). Although the variance of single scattering albedo and surface roughness are not known, the ranges of these parameters reported in literature were used to estimate their variances. The error propagation law is used to compute the standard deviation of the output variable by first generating the outputs using the reference variables and then changing the reference variables by 1% and generating the outputs once gain (Mikhail 1976). The difference between the outputs generated by the 1% change in input variable and the reference input variable is the error used to assess the reliability of the algorithm.

For simplicity the following symbols are used in the derivation of uncertainty matrix.

Inputs (x)
- Single scattering albedo = \( \omega \),
- Surface roughness = \( h \),
- 19GHz vertical = \( V \),
- 19GHz horizontal = \( H \),
- 37GHz vertical = \( k \),

Outputs (y)
- Land surface temperature = \( T \),
- Soil moisture = \( \theta \),
- Vegetation transmissivity = \( \Gamma \).

The output \( y \) is a linear combination of input variables \( x \) and combination coefficient \( A_n \) where \( n \) represents the number of variables \( x \). The combination coefficients are the partial derivatives of the outputs with respect to the inputs.

\[
    y = \begin{bmatrix} T \\ \theta \\ \Gamma \end{bmatrix}
\]

\[
    x = \begin{bmatrix} \omega \\ h \\ V \\ H \\ k \end{bmatrix}
\]

The output \( y \) can be represented as a matrix denoted by;

\[
    y = A^T x
\]

Where \( A^T \) is the transpose of matrix \( A \).

The covariance matrix of \( x \) is given by;

\[
    M(x) = \begin{bmatrix} \omega \omega & h \omega & \omega V & \omega H & \omega k \\ h \omega & h h & h V & h H & h k \\ \omega V & h V & V V & V H & V k \\ \omega H & h H & V H & H H & H k \\ \omega k & h k & V k & H k & k k \end{bmatrix}
\]
It is assumed that there is no correlation between the variables $x$ and therefore the covariance matrix of $x$ reduces to:

$$
M(x) = \begin{bmatrix}
\sigma^2_\omega & 0 & 0 & 0 \\
0 & \sigma^2_h & 0 & 0 \\
0 & 0 & \sigma^2_v & 0 \\
0 & 0 & 0 & \sigma^2_H \\
0 & 0 & 0 & 0
\end{bmatrix} \quad [17]
$$

The covariance matrix of $y$ is given by:

$$
M(y) = A^T M(x) A \quad [18]
$$

And the matrix $A$ is given by:

$$
M(A) = \begin{bmatrix}
\delta T & \delta T & \delta T & \delta T & \delta T \\
\delta \omega & \delta h & \delta V & \delta H & \delta k \\
\delta \theta & \delta \theta & \delta \theta & \delta \theta & \delta \theta \\
\delta \omega & \delta h & \delta V & \delta H & \delta k \\
\delta \Gamma & \delta \Gamma & \delta \Gamma & \delta \Gamma & \delta \Gamma \\
\delta \omega & \delta h & \delta V & \delta H & \delta k
\end{bmatrix}
$$

$$
M(y) = \begin{bmatrix} 
TT & T\theta & T \Gamma \\
\theta T & \theta \theta & \theta \Gamma \\
\theta T & \theta \Gamma & \theta \Gamma
\end{bmatrix} \quad [19]
$$

Although there is no correlation between the input variables, the resultant variances, $M(y)$ are correlated and is expanded to:

$$
M(y) = \begin{bmatrix}
\delta T & \delta T & \delta T & \delta T & \delta T \\
\delta \omega & \delta h & \delta V & \delta H & \delta k \\
\delta \theta & \delta \theta & \delta \theta & \delta \theta & \delta \theta \\
\delta \omega & \delta h & \delta V & \delta H & \delta k \\
\delta \Gamma & \delta \Gamma & \delta \Gamma & \delta \Gamma & \delta \Gamma \\
\delta \omega & \delta h & \delta V & \delta H & \delta k
\end{bmatrix} \begin{bmatrix}
\sigma^2_\omega & 0 & 0 & 0 & 0 \\
0 & \sigma^2_h & 0 & 0 & 0 \\
0 & 0 & \sigma^2_v & 0 & 0 \\
0 & 0 & 0 & \sigma^2_H & 0 \\
0 & 0 & 0 & 0 & \sigma^2_k
\end{bmatrix} \begin{bmatrix}
\delta T & \delta \theta & \delta \Gamma \\
\delta \omega & \delta \omega & \delta \omega \\
\delta \theta & \delta \theta & \delta \theta \\
\delta \omega & \delta \omega & \delta \omega \\
\delta \Gamma & \delta \Gamma & \delta \Gamma \\
\delta \omega & \delta \omega & \delta \omega
\end{bmatrix}
$$

$$
M(y) = \begin{bmatrix}
\delta T & \delta T & \delta T \\
\delta \omega & \delta h & \delta \theta \\
\delta \omega & \delta h & \delta \theta \\
\delta \omega & \delta h & \delta \theta \\
\delta \omega & \delta h & \delta \theta \\
\delta \omega & \delta h & \delta \theta
\end{bmatrix} \begin{bmatrix}
\sigma^2_\omega & \sigma^2_h \\
\delta \theta & \delta \theta \\
\delta \theta & \delta \theta \\
\delta \theta & \delta \theta \\
\delta \theta & \delta \theta \\
\delta \theta & \delta \theta
\end{bmatrix} \begin{bmatrix}
\delta T & \delta T & \delta T \\
\delta \omega & \delta h & \delta \theta \\
\delta \omega & \delta h & \delta \theta \\
\delta \omega & \delta h & \delta \theta \\
\delta \omega & \delta h & \delta \theta \\
\delta \omega & \delta h & \delta \theta
\end{bmatrix}
$$
Note: equation 21 is a 3 x 3 matrix=equation 20. However, due to space limitations, it may not be clear to the reader. Column width ends at $\sigma^2_k$. Equation 21 can be written as:

$$\sigma^2 = \sum_i^m \sum_j^n A_i A_j^T cov_{ij} \delta_{ij}$$  \[22\]

Where $\delta_{ij} = \begin{cases} 0 : i \neq j \\ 1 : i = j \end{cases}$ is the Kronecker delta function and $A$ is the partial derivative of the output variables with respect to input variables.
7.2. Results and discussions

Table 3: Magnitude of errors caused by a 1% uncertainty in the input variables of LST, soil moisture and vegetation transmissivity retrieval

<table>
<thead>
<tr>
<th>1% uncertainty level</th>
<th>Single scattering albedo</th>
<th>Surface roughness</th>
<th>Brightness temperature 19V</th>
<th>Brightness temperature 19H</th>
<th>Brightness temperature 37V</th>
</tr>
</thead>
<tbody>
<tr>
<td>θ(m⁻³)</td>
<td>0.00196</td>
<td>0.00033</td>
<td>0.03708</td>
<td>0.00239</td>
<td>0.04440</td>
</tr>
<tr>
<td>T (k)</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>2.46221</td>
</tr>
<tr>
<td>Γ[-]</td>
<td>0.00072</td>
<td>0.00043</td>
<td>0.05234</td>
<td>0.05091</td>
<td>0.00885</td>
</tr>
</tbody>
</table>

Table 4: Relative contribution of input variables to the total standard deviation of LST, soil moisture and vegetation transmissivity

<table>
<thead>
<tr>
<th>Lat</th>
<th>Lon</th>
<th>out</th>
<th>STD</th>
<th>Single scattering albedo</th>
<th>Surface roughness</th>
<th>TB₁₉V</th>
<th>TB₁₉H</th>
<th>TB₃₇V</th>
</tr>
</thead>
<tbody>
<tr>
<td>30.4°N</td>
<td>92.2°E</td>
<td>T</td>
<td>1.53</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0</td>
<td>0.05</td>
<td>0</td>
<td>0</td>
<td>58.1</td>
<td>3.3</td>
<td>38.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Γ</td>
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<td>0</td>
<td>0</td>
<td>43.6</td>
<td>56.0</td>
<td>0.4</td>
</tr>
<tr>
<td>30.5°N</td>
<td>84.4°E</td>
<td>T</td>
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<td>0</td>
<td>0</td>
<td>0</td>
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<td>100</td>
</tr>
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<td></td>
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<td></td>
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<tr>
<td>34.8°N</td>
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<td>T</td>
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<td>0</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td></td>
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<td>0.5</td>
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<td>50.2</td>
<td>47.2</td>
<td>2.6</td>
</tr>
<tr>
<td>37.0°N</td>
<td>92.5°E</td>
<td>T</td>
<td>1.52</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0</td>
<td>0.02</td>
<td>0</td>
<td>0</td>
<td>57.8</td>
<td>0.1</td>
<td>42.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Γ</td>
<td>0.05</td>
<td>0</td>
<td>0</td>
<td>64.8</td>
<td>34.7</td>
<td>0.5</td>
</tr>
<tr>
<td>32.7°N</td>
<td>84.9°E</td>
<td>T</td>
<td>1.38</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0</td>
<td>0.03</td>
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<td>0</td>
<td>36.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Γ</td>
<td>0.05</td>
<td>0</td>
<td>0</td>
<td>60.8</td>
<td>37.7</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Table 3 above summarizes the magnitude of retrieval errors caused by a 1% uncertainty in the input variables. An increase in the single scattering albedo, surface roughness and the brightness temperature at 19GHz V-polarization causes a decrease in the value of retrieved soil moisture while an increase in the brightness temperatures at 19GHz H-polarization and 37GHz V- polarization causes an increase in the retrieved soil moisture and the opposite holds true. On the other hand the reverse is true for the vegetation transmissivity.
The single scattering albedo and surface roughness contribution to the total standard deviation of the retrieved soil moisture is insignificant. See table 4 above. This finding is similar to the results of Wen, Su et al. (2003). De Jeu (2003) observed that single scattering albedo contributed only 2% and 1.5% to the total standard deviation of the retrieved soil moisture and vegetation optical depth respectively, in the Eurasian sites of his study. De Jeu (2003) also concluded that the influence of surface roughness to the retrieved soil moisture and vegetation optical thickness were negligible.

Error correlation is highest between LST - soil moisture and LST – vegetation transmissivity while the error correlation between soil moisture and vegetation transmissivity is relatively low i.e. 0.43, 0.50 and -0.1 respectively. The correlation is positive except for the soil moisture- vegetation transmissivity. All the correlations are significantly different from zero except for the soil moisture – vegetation transmissivity interaction. Therefore, statistically there is no correlation between the retrieval errors of soil moisture and vegetation transmissivity. However, a statistically significant correlation exists between the retrieval errors of land surface temperature and vegetation transmissivity and land surface temperature and soil moisture. This is largely due to the relatively large retrieval error of land surface temperature in comparison to the retrieval errors of soil moisture and vegetation transmissivity. See equation 21 above.

Soil moisture retrieval using the SSMI is sensitive to the 19GHz and 37GHz vertical polarizations while vegetation transmissivity is sensitive to the 19GHz vertical and horizontal polarization frequencies. Overall the retrieval errors of soil moisture and LST are comparable to other similar algorithms e.g. (Bindlish, Jackson et al. 2003; De Ridder 2003; Wen, Su et al. 2003; Wen, Jackson et al. 2005; Bindlish, Jackson et al. 2008; Draper, Walker et al. 2009)
8. Time series analysis (TSA)

8.1. Theory

Time series analysis is the procedure of fitting a stochastic process to a given time series and it entails three stages: model selection, model estimation and model verification. In this study, a family of TSA methods known as Auto Regressive Integrated Moving Average (ARIMA) (Piwowar and Ledrew 2002; Ford, Goranson et al. 2005) was used to analyze both the land surface temperature and soil moisture time series. A time-series model is characterized by three items; i.e. the orders, which are the numbers of lagged values that appear in the equation, the parameters, which are the associated coefficients and the actual values of the lags, if these differ from the progression 1...n, where n is the number of lags (Castellano-Méndez, González-Manteiga et al. 2004).

An auto-regressive process (AR) is one which can be modelled as a function of the previous observations plus a random error. If $Z_t$ variable measured at time t is a function $\beta$ of the variable measured at an earlier time $Z_{t-1}$ and the variable measured at a much earlier time p, $Z_t$ is expresses as:

$$Z_t = \varepsilon_t + \sum_{i=1}^{p} \beta_i Z_{t-i}$$

Where $t$ is the time of observation, $\beta_1, \beta_2...\beta_p$ are the AR parameters (coefficients), $\varepsilon$ is the random error at time $t$, $p$ is the order of AR and the model is written as AR(p).

A moving average process (MA) on the other hand, is one which can be modelled as a function of the random error of the previous observation. Considering the above example and replacing $p$ with $q$, the variable $Z_t$ is expressed as ;

$$Z_t = \mu + \sum_{i=1}^{q} \gamma_i \varepsilon_{t-i}$$

Where $\gamma_1, \gamma_2...\gamma_q$ are the MA parameters (coefficients), $\varepsilon$ is the random error at a given time, $q$ is the order of the MA and the model is written as MA(q). For a stationary ARIMA model consisting of a non seasonal and seasonal component of the form ARIMA $(p,d,q)(P,D,Q)m$, the model is expressed as;

$$Z_t = \sum_{i=1}^{p} p_i Z_{t-1} + \sum_{i=1}^{q} q_i \varepsilon_{t-1} + \sum_{i=1}^{P} p_i Z_{t-im} + \sum_{i=1}^{Q} Q_i \varepsilon_{t-im} + \varepsilon_t$$

Where $p$ is the order of AR, $d$, is the frequency of non seasonal differencing, $q$ is the order of MA, and $P,D,Q$ and $m$ represent the seasonal order of AR, seasonal frequency of differencing, seasonal order of MA and $m$ is the period of the series which for most seasonal data equal to 12 (Castellano-Méndez, González-Manteiga et al. 2004). For differenced time series, $Z_t$ is replaced with the differenced values $\Delta Z_t$. The steps for the application of an ARIMA model to the time series of SSM/I retrieved soil moisture and land surface are shown in figure 20 and described below.

8.2. Model selection

Soil moisture and LST time series plots for randomly selected 20 pixels were visually inspected and analyzed for properties such as trends, periodicity, changes in the variance, changes in the mean and
presence of outliers in the series. The objectives of this step are three folds: (i) to identify the presence or absence of trends in the environmental variables over time; (ii) to evaluate the need for data transformation required to model the correlation structure in the time series; (iii) to determine the orders of AR and MA in a given time series (Piwowar and Ledrew 2002; Yurekli and Kurunc 2006). Both the plots of soil moisture and LST series have strong seasonal characteristics, with the highest values recorded in summer and the lowest values recorded in winter.

The data was further decomposed into its seasonal component, trend and residuals (figure 21 below). The seasonal and trend decomposition procedure using "loess" (STL), which is a non parametric regression technique was used to decompose the given series by determining the trend using "LOcally wEighted regreSsion Smoother (Loess)" technique, followed by deduction of the trend from the series to obtain the seasonal component and the residuals (Li, Campbell et al. 2003; Zuur and Pierce 2004; Verbesselt, Hyndman et al. 2009). The seasonal decomposition confirmed the presence of a positive trend in both the soil moisture and LST time series. The general “loess” model is given by:

$$Z_t = T_t + S_t + \epsilon_t$$  \[26\]

Where $Z_t$ is the univariate time series, $T_t$ is the trend, $S_t$ is the seasonal component and $\epsilon_t$ is the random error.
Step 1: Model selection

Select 20 random pixels and make time series plots of each.

Analyze all the times series for seasonality, stationarity (trends and heteroscedacity)

Normalize the series for seasonal variation

Plot ACF

Plot PACF

AR or MA & Determine the order of model

Step 2: Model estimation (fitting)

Fit the model to the time series and estimate model parameters using maximum likelihood approach

Evaluate the parameters for statistical significance using standard error of estimate

Test the model for adequacy by comparison of model variance with variance of normalized data and analyze plot of residual ACF for the presence of auto correlation

Evaluate all suitable models using the above process and select the one with best fit

Step 3: Model validation

Plot a time series of residual to check for climatic shift (appear as non stationarity)

Using the model with the best statistical fit, forecast 12 months of surface temperate and soil moisture

RMSE

SSM/I retrieved surface temperate or soil moisture

Figure 20: Schematic illustration of ARIMA modelling of soil moisture and LST
Figure 21: Decomposed soil moisture (m$^3$ m$^{-3}$) [A] and LST(k) [B]
Modelling the correlation structure in a time series is done on a stationary time series, i.e. one with a constant mean and constant variance. In a stationary time series, the covariance between two observations $Z_t$ and $Z_{t+k}$ is a function of the lag $k$ (difference between observation times) of the two observations and is independent of the time $t$ in the series.

Evaluation of a suitable ARIMA model was done through the comparison of sample autocorrelation functions (ACF) and the sample partial autocorrelation functions (PACF) of the soil moisture and LST residuals with the theoretical values (Cimino, Del Duce et al. 1999; Piwowar and Ledrew 2002; Kärner 2009). The ACF is the measure of linear dependence between observations. While the PACF is the measure of the correlation remaining after the auto correlation in time series has been accounted for.

For a given observation $Z_n$, the expectation of $Z_t$ is given by:

$$E(Z_t) = \mu_t$$  \[27\]

And its variance is given by:

$$\sigma^2_t = E(Z_t - \mu_t)^2$$  \[28\]

The auto correlation function can be derived by normalizing auto covariance (dividing by standard deviation) of $Z_t$ which is given by:

$$r(t_1, t_2) = E[(Z_{t1} - \mu)(Z_{t2} - \mu)]$$  \[29\]

$$r(t_1, t_2) = \frac{E[(Z_{t1} - \mu)(Z_{t2} - \mu)]}{\sigma_{t1}\sigma_{t2}}$$  \[30\]

Where $\gamma$ is the covariance of $Z$, $r$ is the auto correlation of $Z$, $\mu$ is the mean of $Z$, $\sigma_{t1}$ and $\sigma_{t2}$ are the standard deviation of $Z$ at time $t_1$ and $t_2$ respectively.

For observations, $Z_1, Z_2, ..., Z_n$, the observations are paired and treated as a bivariate data to estimate the auto correlation between consecutive pairs $(Z_{t1}, Z_{t2}), (Z_{t2}, Z_{t3}), ..., (Z_{tn-1}, Z_n)$ at a given lag $k$ defined by:

$$r_k = \frac{\sum_{i=1}^{n-k}(Z_{t1} - \mu_1)(Z_{t+k} - \mu_2)/n}{\sum_{i=1}^{n}(Z_{t1} - \mu)^2/n}$$  \[31\]

Where $n$ is the length of the time series and $k$ is a given lag (Slini, Karatzas et al. 2002).

Similar to correlation, the auto correlation ranges between -1 and 1 and in the determination of model orders, the ACF and PACF serve to complement each other since they display contrasting characteristics. The ACF plot attenuate for an AR process, while it truncates for a MA process and is not significantly different from zero after lag $q$, meanwhile the PACF plots truncate for an AR process and is not significantly different from zero after lag $p$ and it attenuates for a MA process (Piwowar and Ledrew 2002; Yurekli and Kurunc 2006; Yürekli, Simsek et al. 2007). The characteristic plots are shown in (figures 26 and 27 below). For twenty lags an auto correlation coefficient may fall outside the 95% confidence limit only once by chance.

The ACF is not a valuable time series analysis tool for non stationary time series data. Consequently, seasonal variation and trends in land surface temperature and soil moisture were normalized before
analysis. Achieving stationarity of the time series required two stage approaches, for both land surface temperature and soil moisture to eliminate both the trend and periodicity.

The common methods for de-seasonalizing time series are, seasonal differencing, subtracting monthly averages from the time series (anomalies), periodic function modelling, and the derivation of indicator variables for individual months (Brockwell and Davis 1996; Chatfield 2003). Each of these methods have drawbacks (Zuur and Pierce 2004). However, comparison of methods is outside the scope of this research and therefore a periodic function which has only three parameters as opposed to indicator variables which has twelve parameters was opted for, for this research. A periodic function was first fitted to the data and the resultant residuals were differenced once to obtain stationary soil moisture and temperature residuals for the complete time series analysis (Brockwell and Davis 1996; Castellano-Méndez, González-Manteiga et al. 2004). Tables 5 and 6 below summarize the periodic functions of LST and soil moisture respectively. Figures 22 and 23 below are the plots of LST and soil moisture residuals of periodic functions respectively

The periodic function is given by:

$$Z_t = \beta_0 + \beta_1 \sin\left(\frac{2\pi}{12} t\right) + \beta_2 \cos\left(\frac{2\pi}{12} t\right) + \epsilon_t$$  \[32\]

Where $Z_t$ is the time series variable, $\beta_0$ is the intercept, $\beta_1$ and $\beta_2$ are the coefficients of the sine and cosine functions respectively and $\epsilon_t$ is the random error component of the periodic function.

Table 5: Summary of LST periodic function at location 12

<table>
<thead>
<tr>
<th>coefficient</th>
<th>Estimate</th>
<th>Std error</th>
<th>t-value</th>
<th>P-value</th>
<th>Statistical significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_0$</td>
<td>257.746</td>
<td>0.1596</td>
<td>1615.05</td>
<td>&lt; 0.001</td>
<td>***</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>-8.6144</td>
<td>0.2257</td>
<td>-38.17</td>
<td>&lt; 0.001</td>
<td>***</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>-12.2319</td>
<td>0.2256</td>
<td>-54.21</td>
<td>&lt; 0.001</td>
<td>***</td>
</tr>
</tbody>
</table>

df=243, R^2=0.94, residual std error=2.968

Figure 22: Land surface temperature residuals after eliminating periodicity (location 12)
Table 6: Summary of soil moisture periodic function at location 16

<table>
<thead>
<tr>
<th>coefficient</th>
<th>Estimate</th>
<th>Std error</th>
<th>t-value</th>
<th>P-value</th>
<th>Statistical significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_0$</td>
<td>0.0610</td>
<td>0.0013</td>
<td>48.29</td>
<td>&lt; 0.001</td>
<td>***</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>-0.0642</td>
<td>0.0018</td>
<td>-35.91</td>
<td>&lt; 0.001</td>
<td>***</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>-0.0487</td>
<td>0.0018</td>
<td>-27.23</td>
<td>&lt; 0.001</td>
<td>***</td>
</tr>
</tbody>
</table>

Figure 23: soil moisture residuals after eliminating periodicity (location 16)

The residual plots of the periodic functions of LST and soil moisture clearly indicate that the seasonal signals have been removed. However, the plots are not stationary. For the soil moisture residual plot, the variance is not constant. See figure 23 above. Therefore, first order differencing to eliminate trend and stabilize the variance of the residual data was performed (Castellano-Méndez, González-Manteiga et al. 2004). The general model for first order differencing is given by;

$$\nabla Z_t = Z_t - Z_{t-1} \quad [33]$$

For a given time series with a linear trend of the form;

$$Z_t = \beta_0 + \beta_1 t + \epsilon_t, \quad [34]$$

application of differencing results in a time series with a constant mean, hence stationary time series.

$$\nabla Z_t = (\beta_0 + \beta_1 t + \epsilon_t) - (\beta_0 + \beta_1 (t-1) + \epsilon_{t-1})$$

$$\nabla Z_t = \beta_1 + \epsilon_t - \epsilon_{t-1} \quad [35]$$

Where $\nabla Z$ is the differenced time series variable, $\beta_0$ is the intercept, $\beta_1$ is the slope, $\epsilon$ is the random error and $t$ is the time of measurement.
The differenced stationary residual plots of LST and soil moisture are illustrated in the figures 24 and 25 respectively. The figures clearly demonstrate a stationary time series with a constant mean and constant variance apart from where there are possible breaks in the series.

Figure 24: Land surface temperature residuals after removing both periodicity and trend (location 12)

Figure 25: soil moisture residuals after removing both periodicity and trend (location 16)

The auto correlation and partial auto correlation functions of the stationary residuals were plotted to identify the ARIMA models that describe soil moisture and LST over the Tibetan plateau. This procedure was repeated for 20 randomly selected pixels and the result is summarized in the table 7 below. Three possible models emerged from this preliminary test i.e. ARIMA(0,1,1), ARIMA(1,1,0) and ARIMA(1,1,1). These three models were then used as a guide for an automatic ARIMA model fitting (Hyndman and Koehler 2006; Hyndman and Khandakar 2008). This does not only reveal the seasonal component of model parameters but it also eliminates any subjective decision which could have been made when selecting the model orders. The clear sine wave appearing on the ACF plot in the figure 26 below, is an indication that, there is a seasonal AR component in the time series.
Table 7: ARIMA models for LST and soil moisture derived from the analysis of ACF and PACF

<table>
<thead>
<tr>
<th>Loc No</th>
<th>Lat</th>
<th>Lon</th>
<th>model order</th>
<th>model order</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>30° 14'N</td>
<td>92° 00'E</td>
<td>ARIMA (0,1,1)</td>
<td>ARIMA (0,1,1)</td>
</tr>
<tr>
<td>2</td>
<td>36° 20'N</td>
<td>94° 30'E</td>
<td>ARIMA (1,1,1)</td>
<td>ARIMA (1,1,1)</td>
</tr>
<tr>
<td>3</td>
<td>34° 05'N</td>
<td>99° 58'E</td>
<td>ARIMA (1,1,1)</td>
<td>ARIMA (1,1,1)</td>
</tr>
<tr>
<td>4</td>
<td>38° 05'N</td>
<td>102° 36'E</td>
<td>ARIMA (1,1,1)</td>
<td>ARIMA (1,1,1)</td>
</tr>
<tr>
<td>5</td>
<td>35° 05'N</td>
<td>103° 26'E</td>
<td>ARIMA (1,1,1)</td>
<td>ARIMA (1,1,1)</td>
</tr>
<tr>
<td>6</td>
<td>30° 05'N</td>
<td>98° 13'E</td>
<td>ARIMA (0,1,1)</td>
<td>ARIMA (0,1,1)</td>
</tr>
<tr>
<td>7</td>
<td>28° 11'N</td>
<td>87° 14'E</td>
<td>ARIMA (1,1,1)</td>
<td>ARIMA (1,1,1)</td>
</tr>
<tr>
<td>8</td>
<td>31° 53'N</td>
<td>84° 34'E</td>
<td>ARIMA (1,1,1)</td>
<td>ARIMA (1,1,1)</td>
</tr>
<tr>
<td>9</td>
<td>34° 38'N</td>
<td>84° 48'E</td>
<td>ARIMA (1,1,1)</td>
<td>ARIMA (1,1,1)</td>
</tr>
<tr>
<td>10</td>
<td>37° 17'N</td>
<td>83° 54'E</td>
<td>ARIMA (1,1,1)</td>
<td>ARIMA (1,1,1)</td>
</tr>
<tr>
<td>11</td>
<td>39° 29'N</td>
<td>88° 55'E</td>
<td>ARIMA (1,1,1)</td>
<td>ARIMA (1,1,1)</td>
</tr>
<tr>
<td>12</td>
<td>37° 44'N</td>
<td>93° 40'E</td>
<td>ARIMA (0,1,1)</td>
<td>ARIMA (1,1,1)</td>
</tr>
<tr>
<td>13</td>
<td>36° 17'N</td>
<td>95° 02'E</td>
<td>ARIMA (1,1,1)</td>
<td>ARIMA (1,1,1)</td>
</tr>
<tr>
<td>14</td>
<td>33° 41'N</td>
<td>92° 04'E</td>
<td>ARIMA (1,1,1)</td>
<td>ARIMA (1,1,1)</td>
</tr>
<tr>
<td>15</td>
<td>29° 59'N</td>
<td>87° 21'E</td>
<td>ARIMA (1,1,1)</td>
<td>ARIMA (1,1,1)</td>
</tr>
<tr>
<td>16</td>
<td>34° 17'N</td>
<td>89° 03'E</td>
<td>ARIMA (1,1,1)</td>
<td>ARIMA (1,1,1)</td>
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<tr>
<td>17</td>
<td>31° 05'N</td>
<td>90° 46'E</td>
<td>ARIMA (1,1,1)</td>
<td>ARIMA (1,1,1)</td>
</tr>
<tr>
<td>18</td>
<td>33° 14'N</td>
<td>93° 43'E</td>
<td>ARIMA (1,1,1)</td>
<td>ARIMA (1,1,1)</td>
</tr>
<tr>
<td>19</td>
<td>32° 14'N</td>
<td>97° 10'E</td>
<td>ARIMA (1,1,1)</td>
<td>ARIMA (1,1,1)</td>
</tr>
<tr>
<td>20</td>
<td>35° 41'N</td>
<td>94° 35'E</td>
<td>ARIMA (1,1,1)</td>
<td>ARIMA (1,1,1)</td>
</tr>
</tbody>
</table>

Three final suitable ARIMA models that emerged out of the automatic ARIMA model fitting are; ARIMA(0,1,1) (2,0,0)_{12}, ARIMA(1,1,0) (2,0,0)_{12} and ARIMA (1,1,1) (2,0,0)_{12}. The ACF and PACF plots demonstrated a mixture of AR and MA process for different pixels. See figures 26 and 27 below.
Figure 26: ACF and PACF plots demonstrating a clear AR process of LST at location 12
The ACF plot in figure 26 above attenuates at the low lags up to 5, indicating that the process can be modelled as an AR process and the PACF in the same figure truncates at the first lag suggesting that the order of the AR process is 1. The seasonal AR (sAR) derived from Automatic ARIMA fitting is 2. Recall that the residuals were differenced once. Therefore, for this location the ARIMA model is written as ARIMA (1, 1, 0) (2, 0, 0)$_{12}$. However, in figure 27 the ACF truncates at the first lag indicating that the process can be modelled as a MA and the PACF in the same figure also truncates at the first lag, suggesting that the model incorporates an AR process. In both cases the model order is 1, the residuals were differenced once and the sAR is 2. Therefore, for this location the ARIMA model is written as ARIMA (1,1,1) (2,0,0)$_{12}$ (Piwowar and Ledrew 2002; Yurekli and Kurunc 2006). Similar results were obtained for all the remaining 19 locations for both soil moisture and LST time series.

Figure 27: ACF and PACF plots demonstrating a case of a MA and AR process of soil moisture at location 16. Note; the dotted blue line represents the 95% confidence limits.
8.3. **Model estimation (fitting)**

The above three identified suitable models were fitted to the time series data and the model parameters were estimated. See tables 8 and 9 for soil moisture and LST respectively. The most parsimonious model based on the lowest Akaike Information Criterion (AIC) was evaluated further, for the goodness of fit through the analysis of residual ACF (Akaike 1974; Kim and Kim 2007; Yürekli, Simsek et al. 2007). A well fitted model removes all auto correlation from model residuals and the residual autocorrelations should not be significantly different from zero. See figures 28 and 29 below for soil moisture and LST respectively. Further diagnostic evaluation included test of significance of the parameter estimates and variance of the residuals. The parameters greater than twice their standard error are significant at 95% confidence level (Piwowar and Ledrew 2002).

\[
AIC(m) = n \ln \sigma_a^2 + 2m
\]  

Where \( m \) is the sum of AR and MA orders, \( n \) is the length of the time series and \( \sigma_a^2 \) is the residual variance

<table>
<thead>
<tr>
<th>Model</th>
<th>Parameter</th>
<th>Estimate</th>
<th>Std error</th>
<th>variance</th>
<th>AIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARIMA(1,1,0)(2,0,0)</td>
<td>AR1</td>
<td>-0.3063</td>
<td>0.0629</td>
<td>8.358</td>
<td>1250.72</td>
</tr>
<tr>
<td></td>
<td>sAR1</td>
<td>0.5005***</td>
<td>0.0582</td>
<td>0.00028</td>
<td>-1269.68</td>
</tr>
<tr>
<td></td>
<td>sAR2</td>
<td>0.4520**</td>
<td>0.0591</td>
<td>0.00064</td>
<td>-1053.83</td>
</tr>
</tbody>
</table>

** implies significant at 95% confidence level

<table>
<thead>
<tr>
<th>Model</th>
<th>Parameter</th>
<th>Estimate</th>
<th>Std error</th>
<th>variance</th>
<th>AIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARIMA(0,1,1)(2,0,0)</td>
<td>MA1</td>
<td>-0.3961</td>
<td>0.0891</td>
<td>7.246</td>
<td>1228.95</td>
</tr>
<tr>
<td></td>
<td>sAR1</td>
<td>0.4464***</td>
<td>0.0566</td>
<td>0.00076</td>
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<tr>
<td></td>
<td>sAR2</td>
<td>0.4935**</td>
<td>0.0592</td>
<td>0.00056</td>
<td>-1086.83</td>
</tr>
</tbody>
</table>

** implies significant at 95% confidence level
8.4. Model verification (validation)

Model verification was done through the comparison of the modelled land surface temperature and soil moisture to the satellite derived time series. The coefficient of determination was 94 for soil moisture and 95 for LST. A split sample forecasting was done for both soil moisture and LST time series. The forecast was generated by fitting the suitable models to the first 228 months. This model was then used to forecast the remaining 18 months and the forecasting accuracy assessed (Hyndman and Koehler 2006; Hyndman and Khandakar 2008). Table 10 summarises the results of forecasting. Both the observed and simulated values were within the 95% confidence interval, therefore the model is considered to be valid (Young and Minchin 1991; Piwowar and Ledrew 2002; Romilly 2005). Figures 30 and 31 below show how far into the future soil moisture and LST temperature can be forecast within 80% and 95% confident limits. For soil moisture, the uncertainty band widens significantly within 18 months while the uncertainty band for LST remains relatively stable for over 5 years with a systematic decline.

Figure 28: ACF plot of fitted LST residuals
Figure 29: ACF plot of fitted soil moisture residuals

Figure 30: Long term forecasting of soil moisture at location 16
Figure 31: long term forecasting of Land surface temperature at location 12
Note: the yellow areas represent 95% confident limits while the orange areas represent the 80% confident limits.

Table 10: Summary of soil moisture and LST forecasting accuracy

<table>
<thead>
<tr>
<th>Variable</th>
<th>$R^2$</th>
<th>ME</th>
<th>RMSE</th>
<th>MAE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soil moisture</td>
<td>0.94</td>
<td>-0.033</td>
<td>0.045</td>
<td>0.033</td>
</tr>
<tr>
<td>LST</td>
<td>0.95</td>
<td>2.323</td>
<td>3.052</td>
<td>2.323</td>
</tr>
</tbody>
</table>

ME=mean error, RMSE=root mean square error, MAE=Mean absolute error

In summary, the results of the analysis of the ACF and PACF plots of the fitted models and the comparison of the AIC of the competing models, suggest that soil moisture can be modelled as ARIMA(1,1,0)(2,0,0)$_{12}$ while the LST can be modelled as ARIMA(1,1,1)(2,0,0)$_{12}$. Diagnostic evaluation of the residuals of the fitted models indicated that the residuals were normally distributed with a mean of zero. The residual ACF plot of LST was generally not significantly different from zero. However, some of the residual ACF plot of soil moisture had significant residuals around multiple of 12 suggesting that there is some unexplained variation in the time series. A forecast of both LST and soil moisture provided valid results with an error of 0.045m$^3$m$^{-3}$ for soil moisture and 3K for LST time series.
9. Summary and Conclusions

An algorithm for the retrieval of soil moisture, LST and vegetation transmissivity from the observed brightness temperatures, of the dual polarized 19GHz and 37GHz V-polarized channels of SSM/I, has been applied to retrieve surface variables over the Tibetan plateau. The advantage of this algorithm over other similar algorithms is that, it does not require soil parameters such as soil texture, soil porosity, wilting point or vegetation parameters such as vegetation water content. Soil parameters have insignificant contribution to the total error in soil moisture retrieval (deJeu 2003).

Two assumptions were made in this algorithm; the first assumption was that atmospheric effect on both 19 and 37GHz channels were negligible and the second assumption was that canopy temperature is equal to soil temperature. However, to ensure the second assumption holds true, the SSM/I data for the descending pass of satellites F11 and F13 and the ascending pass of F08 were used in this study. This is because at this time of the day (6:30AM), thermodynamic temperature is relatively stable compared to during afternoon hours and therefore soil temperature can be assumed to be equal to canopy temperature.

A constant value of single scattering albedo and two sets of surface roughness parameters (i.e. 0.05cm for winter and 0.0cm for the rest of the year) were used based on literature and after a validation process to identify suitable parameters for the Tibetan plateau. This study revealed that for the Tibetan plateau surface roughness parameter value of 0.0cm is the most appropriate for soil moisture retrieval. However, the influence of frozen soils on soil moisture retrieval was accounted for by calibrating the roughness parameter during the winter time. This was to counteract the influence of increased emissivity and therefore increased brightness temperature caused by freezing, which impedes the ability of water molecules to rotate.

The retrieved vegetation transmissivity showed seasonal variation with the highest value occurring during the peak of winter and the lowest value occurring during the peak of summer. This is expected because, during the peak of winter, nearly all of the vegetation will have shed off their leaves, hence a higher transparency to microwave emission. On the other hand during the peak of summer nearly all the vegetation have full grown vegetation canopy and therefore less transparent to the microwave emission.

Soil moisture retrieval using the dual polarized 19GHz and the V-polarized 37GHz channels of the SSM/I is sensitive to the 19GHz and 37GHz vertical polarizations while vegetation transmissivity is sensitive to the 19GHz vertical and horizontal polarization frequencies. The time series of both the retrieved land surface temperature and soil moisture are in good agreement with the measured land surface temperature and soil moisture with an error of ±2.87 K and ±0.04 m$^3$ m$^{-3}$ for the land surface temperature and soil moisture respectively.

The entire TP experienced significant warming over the last two decades except the water bodies. However, the central TP experienced more significant positive trend in land surface temperature anomalies. A similar trend is observed over scattered areas in the east and northern part of TP, while the south and south eastern part of the TP experienced the least changes in temperature anomalies. The magnitude of warming over the TP is in the range of 0.2°C to 1.1°C/decade. Warming was more
paramount at elevations over 3000m above sea level compared to areas with elevation below 3000m above sea level. The western and northern part of the TP experienced an increasing significant trend in mean annual soil moisture anomalies. While the eastern and south eastern part of the TP showed no trend in soil moisture anomalies. The soil moisture anomalies over the TP increased at the rate of 0.03 to 0.13% (volume/volume) per year. The annual trend was highest during the summer monsoon season than during the winter season.

LST and soil moisture are crucial in surface energy balance and control of climate feedback mechanism. While LST is well represented in general circulation models (GCMs), there is need to better represent soil moisture in the GCMs to improve their performance. Besides the inclusion of soil moisture in GCMs, prior knowledge of future soil moisture is a prerequisite for better agricultural planning. One of the best ways to understand a stochastic process is to model it. Therefore the time series of both LST and soil moisture were modelled using ARIMA family of time series analysis. As would be expected both LST and soil moisture exhibit strong seasonal characteristics with maximum values being registered in June - July and the minimum values in December – January. ARIMA models were fitted to the data after both the trend and the seasonal components had been removed. Following the analysis of ACF and PACF plots of the fitted models and the comparison of the AIC of the competing models, it is apparent that soil moisture can be modelled as ARIMA(1,1,0)(2,0,0)$_{12}$ while the LST can be modelled as ARIMA(1,1,1)(2,0,0)$_{12}$. Diagnostic evaluation of the residuals of the fitted models indicated that the residuals were normally distributed with a mean of zero. The residual ACF plot of LST was generally not significantly different from zero. However, some of the residual ACF plot of soil moisture had significant residuals around multiple of 12 suggesting that there is some unexplained variation in the time series. A forecast of both LST and soil moisture provided valid results with an error of 0.045m$^{3}$m$^{-3}$ for soil moisture and 3K for LST time series, therefore forecasting using ARIMA model could provide climate models with the needed soil moisture inputs.
10. References:


