A 2D multi-resolution urban flood propagation model using simplified Navier-Stokes equations

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A 2D multi-resolution urban flood propagation model using simplified Navier-Stokes equations

by

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Abstract

This thesis outlines the development of a new 2D Navier-Stokes equations-based flood propagation model suitable for applications in complex urban terrains. The model, which uses an ADI scheme to solve the simplified versions of the equations, can only be applied to fluvial flood events.

The model is applied to Bangkok, Thailand which is situated on the Chao Phraya River deltaic plain. The model was validated using a SAR image-derived flood extent map. The model was able to predict 52% of the inundated area. The model described in this thesis showed a 40% agreement with SOBEK on the flood extent.

The model also showed that open canals help transport the flood away from the river, and in cases like Bangkok, towards the city centre, which would otherwise not have been flooded. The model results suggest that rapid increase in discharge causes more flooding than a gradual one, even if the level of discharge is the same. The model showed most sensitivity towards errors in the Digital Surface Model.
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1. Introduction

1.1. Background

Floods killed more than ten thousand people, displaced 20 million and caused 82 billion US dollars worth of damage in the year 2005 alone [Brakenridge, et al., 2006]. Historically, flooding has superseded all other disasters in terms of frequency, deaths, property destruction, and land degradation [Douben and Ratnayake, 2006]. Floods themselves are a natural phenomenon, but it is human settlement in flood prone areas that make them into disaster. There has been a rapid growth of cities situated in floodplains areas, especially in the developing world [Schultz, 2006]. The future prospects are not heartening: studies point to climate change heightening flood risk [Milly, et al., 2002; Raisanen and Palmer, 2001].

Integrated flood management and flood protection strategies are needed to mitigate the consequences of inundation in urbanised floodplains [Schultz, 2006]. Flood risk assessment forms an integral part of any such strategy, and computer-based modelling is increasingly being used to investigate scenarios [Borrows and de Bruin, 2006]. Conventional one-dimensional (1D) hydrodynamic models are inadequate for simulating wave propagation over complex terrains, such as urban areas [Bates and De Roo, 2000; Molinaro, et al., 1994]. This has led to the development of a slew of two-dimensional (2D) depth averaged models such as TELEMAC2D [Galland, et al., 1991], FLOOD2D [Molinaro, et al., 1994] RMA2 [Norton, et al., 1973], MIKE-21 [Warren and Bach, 1992] and the 1D2D hybrid SOBEK [Stelling, et al., 1998].

Recent advances in the fields of remote sensing and computing have vastly improved data gathering and processing required for flood modelling. This in turn has spawned ever more complex 2D flood models. Unfortunately the total cost of ownership (TCO) for these models is very high as they require expensive licences and even more costly hardware to execute them. These factors necessitate the development of freeware 2D inundation models that can be run on less sophisticated computing resources.

1.2. Problem statement

Whilst existing 2D flood models are good at simulating urban inundation processes, the cost of running them often put them beyond reach of urban authorities in
developing countries. Ironically, it is those places that suffer the most loss of life as a consequence of floods [Zoleta-Nantes, 2002]. This calls for the development of 2D models that are both freeware and executable on low-cost hardware.

Yu and Lane [Yu and Lane, 2006] developed a simplified 2D diffusion-wave based urban fluvial flood model, but admitted that a more sophisticated representation of the inundation process, such as the depth averaged Navier-Stokes equations, is required to effectively simulate the flooding process in an urban terrain.

A myriad of solutions to the Navier-Stokes equations can be found in literature [Bassi, et al., 2005; Feng, et al., 2005; Fidkowski, et al., 2005; Gallagher, et al., 2005; Loukopoulos, 2006; Zhu, 2005], but most of these solutions are only applicable to highly idealised cases such as rectangular reservoirs with cylindrical obstructions. A major challenge in scaling these solutions to model urban flood propagation is the difficulty in representing the complex urban terrain in a form that can be used in the computation, such as a finite-element mesh or a non-uniform Cartesian grid. On the one hand a very fine resolution is required to represent the intricacies of a cityscape, whilst on the other hand, the computational times grow exponentially as the grids or meshes are refined. The main problem here is finding the optimal representation that gives an acceptable compromise between result accuracy and runtime. The matters are further complicated by the fact that urban terrain data are not in homogenous formats; for example the terrain elevation would normally be raster whilst building footprints would be in vector format.

A number of cities world-over are criss-crossed by an intricate network of canals that serve both as drainage systems and transport routes. However, it is poorly understood how canals affect fluvial flood propagation.

One aspect of fluvial urban flooding that is least understood is the relationship between the flood wavelength and spatial and temporal characteristics of the inundation such as water level, flow velocity, inundation duration and flood extent. Lack of such an understanding could weaken the effectiveness of flood mitigation strategies.
1.3.  Research Objectives

1.3.1.  General Objectives

- To develop a 2D Navier-Stokes equations-based flood propagation model suitable for applications in complex urban terrains.
- To investigate the relationship between flood wavelength (shape of the hydrograph) and spatial and temporal characteristics of flood propagation.

1.3.2.  Specific Objectives

- To iteratively simplify 2D Navier-Stokes equations and determine to what degree they could be simplified whilst maintaining acceptable levels of accuracy.
- To develop an optimal representation of the urban terrain in a form that is suitable for use as a model input.
- To study the effect of city canals in the propagation of floods.
- To aim to build the model on top of the widely used PCRaster modelling framework.

1.3.3.  Research Questions

- How should a 2D Navier-Stokes equations-based flood propagation model that can accurately simulate flood propagation over a complex urban terrain be developed?
- What is the relationship between flood wavelength and the propagation characteristics over an urban terrain?
- How should the urban terrain be represented in a form that is suitable for use as a model input?
- What are the effects of canals in urban flood propagation?
- How should the model be built on top of the PCRaster modelling framework?
1.4. Research Hypotheses

Hypothesis 1

$H_0$: There is not a significant relationship between flood wavelength and the propagation characteristics over an urban terrain.

$H_a$: There is a significant relationship between flood wavelength and the propagation characteristics over an urban terrain.

Hypothesis 2

$H_0$: Canals do not significantly alter the flood extent.

$H_a$: Canals significantly alter the flood extent.

Hypothesis 3

$H_0$: Small changes in the urban terrain changes significantly affect inundation characteristics.

$H_a$: Small changes in the urban terrain do not significantly affect inundation characteristics.
2. SWIM-2D: Model formulation

2.1. Navier-Stokes Equations

Navier-Stokes equations, derived from the Newton’s second law of motion, describe the motion of a fluid particle at a given arbitrary point. The motion equation in Cartesian vector notation is

\[
\frac{\partial u}{\partial t} + (u \cdot \nabla)u = -\nabla P + \nu \nabla^2 u
\]

(2.1)

and the continuity of mass is described as

\[
\nabla \cdot u = 0
\]

(2.2)

where \( u \) is the velocity, \( P \) is the pressure divided by density, and the constant \( \nu \) is the kinematic viscosity of the fluid. It is possible to add a body-force term \( F \) to the right hand side of (2.1) to take make the equations take into consideration such effects as gravity, bed friction, wind shear and Coriolis force.

2.2. Shallow Water Equations

The full 3D Navier-Stokes equations applied to large urban domain would be a computationally intensive process whilst it comparisons have shown that 2D depth averaged simplifications are both computationally manageable and provide acceptable results where the flood level is not too high[Quecedo, et al., 2005]. It is possible to further simplify these equations by assuming that viscosity is zero, and the height of the water does not vary during a time step. In the simplified scheme, known as shallow water equation, the flow velocities in orthogonal x and y directions are calculated using the following equations:
\[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + g \frac{\partial h}{\partial x} + uS_x = 0 \]  

(2.3)

\[ \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + g \frac{\partial h}{\partial y} + vS_y = 0 \]  

(2.4)

\[ \frac{\partial h}{\partial t} + \frac{\partial}{\partial x} [u(h - b)] + \frac{\partial}{\partial y} [v(h - b)] = 0 \]  

(2.5)

where \( g \) is the gravitational acceleration constant, \( h \) is the height of the fluid body over the datum, \( b \) is the elevation of the surface over the datum, and \( S_x \) and \( S_y \) are the vertical and horizontal components of bed friction, which can be solved using the manning’s equation as follows:

\[ S_x = gn^2 d^3 \Delta y \sqrt{(v\Delta y)^2 + (u\Delta x)^2} \]  

(2.6)

\[ S_y = gn^2 d^3 \Delta x \sqrt{(v\Delta y)^2 + (u\Delta x)^2} \]  

(2.7)

2.3. Linearised shallow water equations

The advection terms \( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \) and \( u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \) can be ignored if it is assumed that the fluid velocity is small[\textit{Kaas and Miller}, 1990], as is the case of an urban flood plain. This assumption simplifies (2.3) and (2.4) to the following:

\[ \frac{\partial u}{\partial t} + g \frac{\partial h}{\partial x} + uS_x = 0 \]  

(2.8)

\[ \frac{\partial v}{\partial t} + g \frac{\partial h}{\partial y} + vS_y = 0 \]  

(2.9)
Further more, assuming that the depth field is constant over a time step, it is possible to simplify (2.5) as

$$\frac{\partial h}{\partial t} + d \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = 0$$

(2.10)

where depth $d=h-b$

2.4. Discretisation

By spatial discretisation, the following set of equations is obtained from equations 2.10, 2.8 and 2.9 respectively:

$$\frac{\partial h_{i,j}}{\partial t} = \frac{\left( d_{i-1,j} + d_{i,j} \right) u_{i-1,j} - \left( d_{i,j} + d_{i+1,j} \right) u_{i,j}}{2\Delta x}$$  

$$+ \frac{\left( d_{i,j-1} + d_{i,j} \right) v_{i,j-1} - \left( d_{i,j} + d_{i,j+1} \right) v_{i,j}}{2\Delta y}$$

(2.11)

$$\frac{\partial u_{i,j}}{\partial t} = -g \left( \frac{h_{i+1,j} - h_{i,j}}{\Delta x} \right) - uS_x$$

(2.12)

$$\frac{\partial v_{i,j}}{\partial t} = -g \left( \frac{h_{i,j+1} - h_{i,j}}{\Delta y} \right) - vS_y$$

(2.13)

The temporal discretisation is done using the general form

$$\dot{x}^n = \frac{x^{n+1} - x^n}{\Delta t}$$

$$\frac{\partial u_{i,j}}{\partial t} = \frac{u_{i,j}^{n+1} - u_{i,j}^{n}}{\Delta t}$$

(2.14)
from (2.12)

$$-g \left( \frac{h_{i+1,j} - h_{i,j}}{\Delta x} \right) - u_{i,j}'' S_x = \frac{u_{i,j}^{n+1} - u_{i,j}''}{\Delta t}$$

$$u_{i,j}^{n+1} = -g \frac{\Delta t}{\Delta x} (h_{i+1,j} - h_{i,j}) + u_{i,j}'' (1 - S_x)$$

(2.15)

Similarly, discretised forms of \(v\) and \(h\) are obtained thus

$$v_{i,j}^{n+1} = -g \left( h_{i,j+1}^{n+1} - h_{i,j}^{n+1} \right) \frac{\Delta t}{\Delta y} + v_{i,j}'' (1 - S_y)$$

(2.16)

$$h_{i,j}^{n+1} = \Delta t \left[ \left( \frac{d_{i-1} + d_{i,j}}{2\Delta x} \right) u_{i-1,j}^{n+1} - \left( \frac{d_{i,j} + d_{i+1,j}}{2\Delta x} \right) u_{i,j}^{n+1} \right]
+ \frac{d_{i,j} + d_{i,j-1}}{2\Delta y} v_{i,j-1}^{n+1} - \left( \frac{d_{i,j+1} + d_{i,j}}{2\Delta y} \right) v_{i,j}^{n+1} + h_{i,j}''$$

(2.17)

### 2.5. Building the tridiagonal equation system

By substituting (2.15) and (2.16) into (2.17)

$$h_{i,j}^{n+1} = \Delta t \left[ \left( \frac{d_{i-1} + d_{i,j}}{2\Delta x} \right) - g \frac{\Delta t}{\Delta x} \left( h_{i,j}^{n+1} - h_{i-1,j}^{n+1} \right) + u_{i,j-1}'' (1 - S_x) \right]
+ \frac{d_{i,j} + d_{i,j-1}}{2\Delta y} v_{i,j-1}^{n+1} - \left( \frac{d_{i,j+1} + d_{i,j}}{2\Delta y} \right) v_{i,j}^{n+1} (1 - S_y)$$

(2.18)
Equation 2.18 can be transformed thus

\[
\begin{align*}
\frac{\Delta t}{2\Delta x^2} & \left[ \left( d_{i-1,j} + d_{i,j} \right) h_{i,j}^{n+1} \right] \\
& + \left( \frac{d_{i-1,j} + d_{i,j}}{2\Delta x} \right) u_{i-1,j}^{n} (1 - S_x) \\
& + \frac{\Delta t}{2\Delta x^2} \left( d_{i,j} + d_{i+1,j} \right) h_{i,j}^{n+1} - \frac{\Delta t}{2\Delta x^2} \left( d_{i,j} + d_{i+1,j} \right) h_{i,j}^{n+1} \\
& - \left( \frac{d_{i,j} + d_{i+1,j}}{2\Delta x} \right) u_{i,j}^{n} (1 - S_x) \\
& + h_{i,j}^{n+1} \\
\frac{\Delta t}{2\Delta y^2} & \left[ \left( d_{i,j-1} + d_{i,j} \right) h_{i,j}^{n+1} \right] \\
& + \left( \frac{d_{i,j} + d_{i,j-1}}{2\Delta y} \right) u_{i,j-1}^{n} (1 - S_y) \\
& + \frac{\Delta t}{2\Delta y^2} \left( d_{i,j} + d_{i,j} \right) h_{i,j}^{n+1} - \frac{\Delta t}{2\Delta y^2} \left( d_{i,j} + d_{i+1,j} \right) h_{i,j}^{n+1} \\
& - \left( \frac{d_{i,j+1} + d_{i,j}}{2\Delta y} \right) u_{i,j}^{n} (1 - S_y) \\
+ h_{i,j}^{n+1}
\end{align*}
\]

This can be further simplified as:
A 2D MULTI-RESOLUTION URBAN FLOOD PROPAGATION MODEL USING SIMPLIFIED NAVIER-STOKES EQUATIONS

\[
\begin{align*}
    h_{n+1}^{i,j} &= \Delta t \left\{ -\frac{g\Delta t}{2\Delta x^2} (d_{i-1,j} + 2d_{i,j} + d_{i+1,j}) h_{n+1}^{i,j} + \frac{g\Delta t}{2\Delta x^2} (d_{i-1,j} + d_{i,j}) h_{n}^{i,j-1} \\
    &+ \left( \frac{d_{i+1,j}}{2\Delta x} \right) u_{n}^{i,j-1} (1 - S_y) \\
    &+ \frac{g\Delta t}{2\Delta y^2} (d_{i,j} + d_{i+1,j}) h_{n+1}^{i+1,j} - \left( \frac{d_{i,j} + d_{i+1,j}}{2\Delta y} \right) v_{n}^{i,j} (1 - S_x) \\
    &+ \frac{g\Delta t}{2\Delta y^2} (d_{i,j+1} + d_{i,j}) h_{n+1}^{i,j+1} - \left( \frac{d_{i,j+1} + d_{i,j}}{2\Delta y} \right) v_{n}^{i,j+1} (1 - S_y) \right\} \\
    &= \Delta t \left\{ \frac{g\Delta t}{2\Delta x^2} (d_{i-1,j} + 2d_{i,j} + d_{i+1,j}) h_{n+1}^{i,j} + \frac{d_{i-1,j} + d_{i,j}}{2\Delta x} u_{n}^{i,j-1} (1 - S_x) \\
    &+ \frac{d_{i,j} + d_{i+1,j}}{2\Delta y} v_{n}^{i,j} (1 - S_x) \\
    &+ \frac{d_{i,j} + d_{i,j+1}}{2\Delta y} v_{n}^{i,j+1} (1 - S_y) \right\} \\
    &+ h_{n}^{i,j}
\end{align*}
\]

The terms can be arranged thus:

\[
\begin{align*}
    \begin{bmatrix}
        \begin{array}{c}
            d_{i-1,j} + 2d_{i,j} + d_{i+1,j} \\
            d_{i,j} + d_{i+1,j}
        \end{array}
    \end{bmatrix}
    \begin{bmatrix}
        \begin{array}{c}
            \frac{\Delta t}{2\Delta x^2} \\
            \frac{\Delta t}{2\Delta y^2}
        \end{array}
    \end{bmatrix}
    + 1
    \begin{bmatrix}
        h_{n+1}^{i,j} \\
        h_{n+1}^{i+1,j}
    \end{bmatrix}
    \begin{bmatrix}
        \frac{\Delta t}{2\Delta x^2} (d_{i-1,j} + d_{i,j}) h_{n+1}^{i,j} + \frac{d_{i-1,j} + d_{i,j}}{2\Delta x} u_{n}^{i,j-1} (1 - S_x) \\
        \frac{d_{i,j} + d_{i+1,j}}{2\Delta y} v_{n}^{i,j} (1 - S_x) \\
        \frac{d_{i,j} + d_{i,j+1}}{2\Delta y} v_{n}^{i,j+1} (1 - S_y) \end{bmatrix}
    + \Delta t
    \begin{bmatrix}
        h_{n}^{i,j}
    \end{bmatrix}
\end{align*}
\]

\[2.19\]
2.6. ADI Scheme

In Equation 2.19, it is possible to identify two sets of tridiagonal equations in the general form:

\[ a(i)t(i) = b(i)t(i + 1) + c(i)t(i - 1) + d(i) \]

One set contains the terms \( h_{i,j}^{n+1} \), \( h_{i+1,j}^{n+1} \) and \( h_{i-1,j}^{n+1} \) whilst the other contains the terms \( h_{i,j}^{n+1} \), \( h_{i,j+1}^{n+1} \) and \( h_{i,j-1}^{n+1} \).

Therefore it is possible to solve this using an Alternating Direction Implicit (ADI) scheme, where by the equations are solved sequentially in the x and y directions in two half-time steps \( n+\frac{1}{2} \) and \( n+1 \) respectively.

![Figure 1: ADI Scheme](image)

At \( t = n + \frac{1}{2} \) values for \( h_{i,j}^{n+1} \) and \( h_{i,j-1}^{n+1} \) are approximated to values for \( h_{i,j+1}^{n} \) and \( h_{i,j}^{n} \).

At \( t = n + 1 \) values for \( h_{i+1,j}^{n+1} \) and \( h_{i-1,j}^{n+1} \) are approximated to values for \( h_{i+1,j}^{n+\frac{1}{2}} \) and \( h_{i,j}^{n+\frac{1}{2}} \).

Using this technique, two intermediate sets of equations are obtained thus...
\[
\frac{g \Delta t^2}{2 \Delta x^2} \left( \frac{d_{i-1,j} + 2d_{i,j} + d_{i+1,j}}{2 \Delta x^2} \right) h_{i,j}^{n+\frac{1}{2}} \\
+ 1
\]

\[
= \frac{g \Delta t^2}{2 \Delta x^2} \left( d_{i-1,j} + d_{i,j} \right) h_{i-1,j}^{n+\frac{1}{2}} + \frac{g \Delta t^2}{2 \Delta x^2} \left( d_{i,j} + d_{i+1,j} \right) h_{i+1,j}^{n+\frac{1}{2}}
\]

\[
+ \Delta t \left[ \frac{\left( \frac{d_{i,j} + d_{i-1,j}}{2 \Delta x} \right) u_i^n (1 - S_x)}{2 \Delta x} - \left( \frac{d_{i,j} + d_{i+1,j}}{2 \Delta x} \right) u_i^n (1 - S_x) \right]
\]

\[
+ \Delta t \left[ \frac{\left( \frac{d_{i,j} + d_{i,j-1}}{2 \Delta y} \right) v_{i,j}^n (1 - S_y)}{2 \Delta y} + \left( \frac{d_{i+1,j} + d_{i,j}}{2 \Delta y} \right) v_{i,j}^n (1 - S_y) \right] + h_{i,j}^n
\]

(2.20)

and
The algorithm for modelling the fluid motion is given below:

1. Initialise \( h(0), u(0), v(0), \) and \( b \)
2. Loop for all timesteps \( t \rightarrow \)
   
3. Solve \( h_{i,j}^{n+1/2} \) using (2.20)
4. Solve \( h_{i,j}^{n+1} \) using (2.21)
5. Update \( u_{i,j}^{n+1} \) and \( v_{i,j}^{n+1} \) using (2.15) and (2.16) respectively
6. End Loop
2.8. **Optimisation: Dry cell exclusion**

For the sake of computational efficiency, the model only ignores dry cells that are also completely contained by dry cells. This speeds up the running of the model when only a relatively small area is flood, as would be the case during the beginning of the flood. However, as the flood propagates, more and more cells become wet which in turn slows down the running time of the model.

![Dry cell exclusion diagram](image)

**Figure 2: Dry cell exclusion**

2.9. **Simultaneous Multi-resolution Modelling**

The difference between traditional approaches and SWIM-2D is that this model is able at different resolutions simultaneously. Figure 3 shows a typical multi-resolution setup.

2.9.1. **Translations between resolutions**

When moving from a large cell to smaller cell, the water height $h$ and the velocity vector components $u, v$ will be the same for all the smaller cells covered by the bigger cell.

Going from a smaller cell to a bigger cell is more complex. The height $h$ of the bigger cell is taken to be the average of all the smaller cells covered by the bigger cell. The velocity vector components $u, v$ is taken to be the $u, v$ of the small cell
closes to the boundary with the big cell. If there’s more than one such cell, the one is chosen by random.

Figure 3: Multi-resolution grids
3. Model Testing

3.1. Suitability of Study Area

The model was simplified on the grounds of three major assumptions, namely:

1. The viscosity of the water is zero
2. Fluid velocity is small
3. The height of the water does not vary during a time step

The first assumption rules out the applicability of this model to a mud flow, or water carrying a large amount of sediment. The model is not suitable for gravity driven fluid flow down a very steep gradient, as would be expected in a mountainous catchment area. The model should also not be used in cases where the flood depth to flood extent ratio is small, as is the case would be in a ‘damn break’ scenario for a small reservoir.

Given the above, the ideal study area for a simplified model is an urban watershed in a deltaic flood plain.

3.2. Required Data

The model would require the data outlined in Table 1 as inputs.

<table>
<thead>
<tr>
<th>Table 1 Required Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
</tr>
<tr>
<td>Digital Elevation</td>
</tr>
<tr>
<td>Model (Raster)</td>
</tr>
<tr>
<td>Building footprints</td>
</tr>
<tr>
<td>(Vector)</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Inflow discharge</td>
</tr>
<tr>
<td>hydrograph</td>
</tr>
<tr>
<td>Land use map</td>
</tr>
<tr>
<td>Manning’s</td>
</tr>
</tbody>
</table>
3.3. Study Area

<table>
<thead>
<tr>
<th>River profile</th>
<th>Surveyed cross section</th>
<th>Required for calibration/validation. SAR can be used to segregate flooded and non-flooded areas [Sanyal and Lu, 2004].</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flood extent of a past flood</td>
<td>SAR Image</td>
<td></td>
</tr>
</tbody>
</table>

ASTER Image (April 06 2005)
Bangkok, the political, commercial, and cultural capital of Thailand is located in the low floodplains of the Chao Phraya River, about 25km from the mouth. It is elevation varies between 0m and 1.5m above mean sea level[Sinsakul, 2000]. “Krungthep Maha Nakhorn”, as it is called locally, is home to six million residents and it is estimated that four million more people commute into the city daily from the suburbs [National Statistics Office, 2000].

Since its establishment as the Capital of Thailand (then known as Siam) in 1782, Bangkok has been inundated repeatedly. The last major flood events in the city occurred in 1975, 1980, 1982, 1983, 1995 and 1996 [Atthanand Suphat, 2001]. The flood risk for densely populated areas continue to heighten due to a combination of the gradient of the lower plain decreasing [Haruyama, 1993] and land subsidence[UNEP, 2001]. Whilst the authorities have attempted to contain the threat through hard engineering, the overall flood risk has increased due to flood waters reaching elevation levels more rapidly[The Working Group of the Office of Natural Water Resources Committee (ONWRC) of Thailand, 2003].

Bangkok was chosen as the study area for a number of reasons. Firstly, Bangkok is a so-called “monsoon megacity” located in right in the middle of a floodplain. The Thai capital represents a densely urbanised terrain with a significant portion of the land area being occupied by buildings, roads and other impermeable surfaces. The city is criss-crossed by a series of canals, making the landscape interesting from a hydrological point of view.

3.4. DSM Construction

The Digital Surface Model (DSM) to be used as an input was provided by another researcher[Chen, 2007]. Chen constructed the DSM by combining the basic data: processed DEM, land use map, road network and canal network. Buildings were given an arbitrary high value of 5m.
Figure 4 DSM [Chen, 2007]

The DSM is 453 pixels by 921 pixels with a resolution of 30m.

3.4.1. Channel Adjustment

A major limitation with a purely 2D model is the handling of narrow canals. A canal appearing as in the left hand image of Figure 5 would not be continuous. Therefore, a small amount of corrections were needed to ensure that the flow was not obstructed, as shown on the right hand image in Figure 5.
3.5. Simulations

Three sets of simulations were carried out: first to validate the model, second to answer the research questions and finally to analyse the sensitivity.

<table>
<thead>
<tr>
<th>Simulation</th>
<th>Resolution</th>
<th>Simulation Length (Steps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1995 Flood</td>
<td>Single</td>
<td>100 Days (864000)</td>
</tr>
<tr>
<td>1995 Flood</td>
<td>Multiple</td>
<td>100 Days (864000)</td>
</tr>
<tr>
<td>Comparison with SOBEK</td>
<td>Single</td>
<td>5 Days (43200)</td>
</tr>
<tr>
<td>Canals open</td>
<td>Single</td>
<td>5 Days (43200)</td>
</tr>
<tr>
<td>Canals closed</td>
<td>Single</td>
<td>5 Days (43200)</td>
</tr>
<tr>
<td>Different flood wavelengths</td>
<td>Single</td>
<td>100 Hours – 12 Hours (36000 – 4320)</td>
</tr>
<tr>
<td>Sensitivity Analysis</td>
<td>Single</td>
<td>1 Day (25920)</td>
</tr>
</tbody>
</table>

3.5.1. Discharge

Before each of the simulations, the river was filled up gradually until a stable water level was reached using a linear increase in discharge. For the 1995 simulation, actual values were used as shown in Figure 6.
3.5.2. Sensitivity Analysis

Sensitivity of the model to the variations in input parameters was assessed. For this analysis the DSM, Discharge, and Friction values were changed by +10-10% uniformly, and then the changes in the flood extent was calculated.
4. Results

4.1. Evaluation criteria

Model was evaluated by comparing the flood extent obtained by SAR image \( A_{\text{obs}} \) against the extent predicted by the model \( A_{\text{mod}} \), and then evaluating the percentage measure of fit [Bates and De Roo, 2000] using the equation below:

\[
\text{Fit} = \frac{A_{\text{obs}} \cap A_{\text{mod}}}{A_{\text{obs}} \cup A_{\text{mod}}} \times 100
\]

4.2. 1995 Flood event

The model only achieved a fit of 27.12% with the SAR data in the single resolution mood, as there were significant errors at the northern boundaries. However, the model achieved a 52.92% fit in the multi-resolution mode, as there were no significant errors at the boundary. Furthermore, there was a water balance error of 6.4% of total inflow over simulation period.
4.3. Comparison with SOBEK

The other researcher [Chen, 2007] supplied the results of a SOBEK run for 10 year return period run. The model was run with almost the same data set as that used by Chen. Only the single resolution mode was used to make as SOBEK only works in one resolution simultaneously. Figure 8 shows the results after 24 hours.
The maximum flood extent of the two flood models were compared using the best fit approach and it was found that there was a 40.23% agreement between the models.

4.4. **Effect of Canals**

Model simulations showed that when canals are open during a river flooding even, flood water travels along the canals and causes flooding in areas that would otherwise not be flooded by the same discharge. With the canals open, it is possible to see how the flood propagates through the city core, as show in Figure 9.
4.5. **Effect Floodwave length**

Model simulation showed that more flooding occurred even with the same level of discharge if the discharge increased more rapidly. When the time it took reach the maximum flood level was halved, the flood extent increased by 14.6%. It must be noted that the increase in the flood extent happened even though the total inflow volume halved.

However, the model itself underperformed when there was a steeper curve on the hydrograph. The mass balance error went up to 9.3%.
4.6. Sensitivity Analysis

Of the three variables considered the model showed most sensitivity to changes in the DSM, followed by sensitivity to inflow hydrograph. The model showed negligible changes to Manning’s $n$. 

Figure 10 Flood wavelengths
5. Discussion

5.1. Model Ability

The simulated flood extent of SWIM-2D did not show a satisfactory agreement with the flood extent derived from the SAR image. This in itself does not mean the model is wrong and the SAR image as there is a degree of uncertainty when estimating flood extent using SAR imagery [Horritt, 1999]. The mass balance errors of the model are acceptably low under normal flow situations. The per cell balance error was negligible even though the accumulated mass balance error amounted to more than 6% of the total inflow volume. The model was stable for all available data relating to historical hydrographs.

Running the model in multi-resolution mood created a buffer around the area of interest thereby moving the problems related with boundaries away from the immediate area of concern. While being computationally more efficient, this method has the added advantage that crude low resolution outer section of the domain does not require the same level of data quality as our focus area.

5.2. Comparison with SOBEK

The situation in comparison to SOBEK is also similar with only a 40% agreement between the two models. SOBEK is a 1D-2D model, whereas SWIM-2D is purely 2D model. Most of the discrepancies in the results can be found near the narrow channels, and it can be argued that 1D-2D solution outperforms a purely 2D solution as it is more accurate to represent a narrow channel in 1D. Furthermore, the discharge hydrograph used for comparing the two models is somewhat unnatural. In the hydrograph used for the SOBEK the water discharge varies from 500 m$^3$s$^{-1}$ to 4500 m$^3$s$^{-1}$ within the space of 48 hours. A rapid increase of this nature represents a hydraulic shock for SWIM-2D, and it affected the stability of the model. Unfortunately, it was not possible on this occasion to run longer period simulations on SOBEK as a simulation of five days takes equal amount of time to execute.
5.3. **Effect of Canals**

The canals, if left open during a river flood event, act as a conduit for moving the floodwater to other parts of the city. While canals help to prevent pluvial flooding, they exacerbate fluvial flooding. SWIM-2D currently has no rainfall induced flood modelling component, but an extension of the model to include such features would enable the study of canals under a combined fluvial-pluvial flooding situation.

5.4. **Effect Floodwave length**

The model results show that the flood wavelength has an impact on the characteristics of the flood propagation. The hydrograph shape induced by a short sharp storm exceeds the flood extent of a longer drawn out flooding event, as would be caused by the monsoon. However, SWIM-2D itself showed greater instability when there is a rapid change in discharge.

5.5. **Sensitivity Analysis**

In terms of sensitivity the DSM shows the most sensitivity. Therefore, errors in the DSM have a major effect on the overall model ability. The model is also significantly sensitive to the inflow hydrograph. The model showed negligible sensitivity to the changes in Manning’s n. This suggests that using values from literature should be sufficient for the friction co-efficient.
6. Conclusions

This thesis sought to develop a 2D Navier-Stokes equations-based flood propagation model suitable for applications in complex urban terrains. The equations were simplified and solved using an ADI method. The study also looked at the suitability of a simultaneous multi-resolution scheme. The 1995 flood simulation agreed in general with the observed SAR data, but it is difficult to give an exact estimate of the models accuracy as flood shorelines obtained from SAR imagery carries an inherent level of uncertainty.

The model fitted reasonable well with the SOBEK predictions. The 1D2D models are better at representing narrow channels such as lakes and small streams, and most of the discrepancies between the two models appear near narrow channels.

The model showed that the canals can help flood propagation during a fluvial flood. The effect of canals on a fluvial-cum-pluvial flooding event could be a part of a further study. The model results indicated that a rapid increase in discharge causes a bigger flood than a gradual increase in discharge.

The model showed most sensitivity to the DSM, suggesting that during the data collection, the focus of attention should be collecting accurate urban terrain data.
References

Brakenridge, G. R., et al. (2006), 2005 Flood Archive, edited, Dartmouth Flood Observatory, Hanover, USA.


